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CLÉMENT COLSON

PAR RENÉ ROY

LE 24 MARS 1939, s'éteignait à Paris Clément Colson, membre de la Société Internationale d'Économétrie dès la fondation de ce groupement. A celui qui fut notre maître et nous honora de son amitié, nous aurions aimé rendre un témoignage digne de sa haute personnalité, mais les circonstances ne nous ont pas permis de rassembler tous les documents nécessaires; nous voulons cependant espérer que les lecteurs de la revue ECONOMETRICA excuseront la sècheresse de cette note. L'essentiel à notre sens est de marquer la place occupée par Clément Colson dans la science économique, l'enseignement et le domaine de l'action, car ce fut à la vérité dans ces trois directions qu'il exerça ses dons exceptionnels.

Né le 13 novembre 1853 à Versailles, il fut admis en 1873 à l'École Polytechnique; il y fut le condisciple d'Henri Poincaré en qui élèves et maîtres se plaisaient à reconnaître déjà l'infaillible signe du génie mathématique. Classé dans le corps des Ponts et Chaussées, Clément Colson acquit ensuite à notre vieille école de la rue des Saints-Pères, les connaissances techniques, propres à le familiariser avec l'art et la langue des ingénieurs; cette formation n'est certainement pas étrangère à ce sens du concret qui caractérise toute son œuvre. Mais il n'entendait pas embrasser la carrière d'ingénieur et, dès sa sortie de l'École des Ponts et Chaussées, il prépara le concours d'Auditeur au Conseil d'État. Ayant satisfait à cette épreuve, il poursuivit sa carrière administrative au sein de la Haute-Assemblée; en 1923, il en devint le Vice-Président et y demeura jusqu'en 1928, époque à laquelle il prit sa retraite.

Ce n'est pas seulement au Conseil d'État que devait s'exercer l'action administrative de Clément Colson: d'abord sous-chef, puis Chef de Cabinet auprès du Ministre des Travaux Publics, il devient en 1885 l'Adjoint d'Alfred Picard, Directeur Général des Chemins de fer, des Routes, de la Navigation et des Mines. En 1894, il est nommé Directeur des Chemins de fer et se signale à la fois par ses qualités de décision, de haute compétence et aussi par son indépendance à l'égard des hommes politiques. Il devait par la suite consacrer une large part de son action et de sa pensée aux diverses questions économiques intéressant les travaux publics, spécialement l'exploitation des chemins

de fer, celle des Ports et la navigation maritime. Par ses écrits, dont plusieurs parurent aux *Annales des Ponts et Chaussées*, par ses interventions dans les Commissions administratives ou les Congrès, il s'efforça constamment de faire prévaloir ses vues générales en proposant des solutions pour les problèmes particuliers que ses fonctions lui avaient donné l'occasion d'étudier. Jusqu'à la fin de sa carrière, il participa notamment aux travaux du Congrès international des chemins de fer où sa compétence était justement appréciée.

C'est au contact de ces réalités que se dessinèrent à la fois sa carrière scientifique et son activité professorale: il y débute en 1885 par un cours à l'École des Hautes Études Commerciales et par la publication en 1890 de son volume *Transports et Tarifs*. En 1892, il est nommé Professeur à l'École des Ponts et Chaussées, où il rénove l'enseignement économique. En 1909, il devient Professeur à l'École libre des Sciences Politiques. De 1901 à 1907, il publie son *Cours d'Économie Politique* qui, en 1910 lui ouvre les portes de l'Académie des Sciences morales et politiques. Ce cours, incorporé dans l'encyclopédie des travaux publics, comprend l'enseignement général de l'économie politique avec applications aux finances et aux travaux publics; conçu pour un auditoire à formation mathématique, il s'attache essentiellement à la notion de valeur et corrélativement à l'incidence de la législation, de la fiscalité, des mesures douanières, des coalitions, sur les prix, le niveau des salaires et le taux de l'intérêt. Nommé en dernier lieu Maître de Conférences à l'École Polytechnique au mois de juin 1914, Clément Colson y développe son enseignement économique et social après la guerre de 1914–1918. Il devait conserver la majeure partie de ces cours jusqu'à la fin de sa carrière.

A ces multiples activités, s'en ajoutaient bien d'autres. Nous en donnerons quelque idée par les exemples suivants:

Il crée en 1896, à la *Revue Politique et Parlementaire*, une chronique trimestrielle des "Questions de Transport" à laquelle il s'est consacré jusqu'en 1932.

A la Société d'Études législatives, il rédige en 1907, sur les Conventions collectives, un rapport magistral; l'année suivante, il accède à la présidence de cette Assemblée.

Dans son livre *Organisme économique et désordre social*, publié en 1912 dans la Bibliothèque de philosophie scientifique, il aborde les problèmes de l'ordre moral qu'il reprendra plus tard dans le tome VII de son *Cours*.

Après la guerre de 1914–1918, il se fit un devoir de remettre complètement à jour tous les tomes de son *Cours;* il y parvint avant de prendre sa retraite.

En matière de statistique, son activité fut particulièrement marquée,

puisqu'il devint successivement Membre de l'Institut International de Statistique en 1906, Président du Conseil de la Statistique Générale de la France de 1918 à 1936, Vice-Président du Conseil Supérieur de Statistique en 1920, Vice-Président du Conseil de l'Institut de Statistique de l'Université de Paris en 1922, Président de la Société de Statistique en 1929.

Par la nature même de ses fonctions, Clément Colson devait en premier lieu s'attacher aux problèmes relevant de l'économie publique; mais loin de se cantonner dans ce domaine déjà très vaste, il marqua également son empreinte en économie politique générale.

Parmi les travaux concernant l'économie publique, les plus importants paraissent bien être ceux qui ont trait à l'exploitation des monopoles; en cette matière, son apport est double:

Il démontre tout d'abord la supériorité de la concession sur l'exploitation en régie et pour ce faire, il use non seulement d'arguments théoriques, mais aussi de la comparaison des résultats obtenus par l'un et l'autre mode d'exploitation. Ses études sur la concession de chemins de fer du type français et la régie du type allemand ont pleinement résisté aux controverses qu'elles ont suscitées; par la maîtrise du raisonnement aussi bien que par la connaissance des faits, elles constituent même un modèle du genre.

Outre ce choix entre les deux modes d'exploitation, il s'est attaché à l'étude des clauses financières qui régissent les concessions de services publics.

D'autre part il applique systématiquement la théorie de Dupuit sur les péages et il en pousse l'analyse dans le détail pour définir les principes sur lesquels doit reposer toute tarification commerciale; pour les chemins de fer en particulier, il est ainsi conduit à serrer de très près le problème de la distinction entre les charges permanentes et le prix de revient partiel. Il s'affirme également le continuateur de Dupuit lorsqu'il recourt à la notion de l'utilité des travaux publics pour établir un parallèle entre l'économie de la voie d'eau et celle de la voie ferrée.

Ce sont toujours les mêmes principes réalistes qui l'inspirent lorsqu'en économie politique générale, il élabore sa théorie du salaire que l'on pourrait désigner théorie de la détermination conjointe du salaire et du taux de l'intérêt, car il en conçut le mécanisme en procédant à une analyse minutieuse des facteurs à considérer pour le perfectionnement des installations ferroviaires: en l'espèce, la décision à prendre se trouve en effet commandée par la comparaison entre les charges de capital liées au taux de l'intérêt et la réduction des frais de main-d'œuvre, liée au niveau des salaires. A la vérité, Walras avait déjà noté l'interchangeabilité de la force de travail et des machines, mais

c'est bien à Clément Colson que revient le mérite d'avoir véritablement édifié une théorie qui met clairement en lumière l'étroite dépendance du niveau des salaires et du taux de l'intérêt.

Là ne se borne pas certes l'apport de Clément Colson à l'économie pure et appliquée; certaines recherches, comme l'inventaire de la richesse en France, donnent à la fois des résultats et une méthode d'évaluation en une matière qui jusqu'alors n'avait été que très imparfaitement traitée. Mais nous n'avons pas dessein d'épuiser la série de ses recherches et nous entendons strictement nous limiter à l'essentiel. De ce point de vue, il est intéressant d'observer qu'à l'opposé des fondateurs de l'équilibre économique, Clément Colson, sollicité par des mobiles d'ordre concret, suit la voie tracée par Cournot, Dupuit et Marshall, en ce sens que ses théories ne visent qu'à la réalisation d'équilibres partiels. Telle est sans doute la conception à laquelle doit s'arrêter le chercheur qui, désireux de résoudre des problèmes concrets, entend néanmoins parvenir à ces principes généraux et abstraits qu'impliquent toutes les élaborations de la science.

L'œuvre aussi bien que l'enseignement de Clément Colson demeurent presque exempts de symboles mathématiques; de fait, il confessait qu'assez vite, il avait cessé de recourir à cette forme de langage. Son esprit réaliste se refusait d'ailleurs à modeler strictement les phénomènes concrets sur les modèles abstraits; mais tout en assignant des limites à l'emploi du langage algébrique, il ne manquait pas d'observer combien l'école mathématique avait puissamment contribué à l'édification des théories de la valeur. Il usait fréquemment de représentations graphiques et la revue ECONOMETRICA[1] publia ces dernières années la réponse de Marshall à la lettre que lui adressa Clément Colson pour s'excuser d'avoir ignoré son œuvre et de ne pas en avoir fait mention.

S'il n'usait pas de symboles, notre auteur est pourtant resté fortement attaché à sa formation mathématique, car point n'est besoin de recourir explicitement à l'expression algébrique pour se ranger parmi les adeptes de cette discipline. Le plus important n'est-il pas en effet d'asseoir le raisonnement sur les concepts empruntés à la mathématique, tels que les notions de variable indépendante, de liaison fonctionnelle, de fonction continue, de maximum ou de minimum, d'équilibre stable, etc. . . ? Or, il suffit d'avoir simplement parcouru l'œuvre de Clément Colson pour se pénétrer de l'importance que tiennent ces notions dans ses écrits. Négligeant la lettre pour n'en retenir que l'esprit, nous sommes ainsi pleinement fondé à compter Clément Colson parmi les économistes mathématiciens.

[1] Vol. 1, April, 1933, pp. 221–222.

De même, s'il n'a pas utilisé les méthodes raffinées de la statistique moderne, il a néanmoins recouru systématiquement à la documentation statistique pour l'exposé de ses théories et pour leur illustration par les faits. Mais il refusait de se laisser asservir par un outil qui, manié sans discernement, devient trop pesant pour être d'un usage vraiment fécond; aussi a-t-il constamment dominé les éléments statistiques en les soumettant à une critique rigoureuse et en rejetant ceux qui n'offraient pas suffisamment de garantie. C'est là sans aucun doute la raison pour laquelle ses écrits présentent l'incomparable mérite de n'être pas alourdis par ce matériel surabondant qui, chez tant d'auteurs, ne fait qu'obscurcir la pensée.

Qu'il rédige, qu'il enseigne ou qu'il agisse, Clément Colson s'est d'ailleurs préoccupé avant tout de parvenir à la vérité, car la droiture et la probité intellectuelle constituaient les traits dominants de sa personne.

Sur le terrain doctrinal, Clément Colson resta le champion déterminé du libéralisme et contrairement à certaines appréciations superficielles, ce ne fut aucunement le dogmatisme, mais la méditation et le commerce des hommes qui le maintinrent dans cette tradition classique de la pensée économique. Loin d'être emporté par le courant interventionniste qui depuis le début du siècle gagna les sphères influentes de la politique, de l'enseignement et même des affaires, il s'efforça toujours de réagir en s'appuyant sur l'inébranlable foi qu'il avait dans la puissance du raisonnement fondé sur l'observation. Convaincu avant tout que les lois économiques présentaient le même caractère de nécessité que les autres lois scientifiques, il ne se laissait pas abuser par les mérites apparents ou les facilités de l'interventionnisme socialisant. Il savait également résister à la séduction des arguments inspirés d'une fausse conception de la sensibilité; l'ouvrage qu'il publia en 1912 sous le titre *Organisme économique et désordre social* renferme à ce sujet bien des pages que chacun d'entre nous pourrait utilement méditer à la lumière des évènements actuels. Certains de ses adversaires ont eu parfois tendance à l'accuser de sècheresse en prenant justement prétexte de la réserve qu'il s'imposait en matière de sensibilité. Ceux qui l'ont bien connu, ceux qu'il a estimés, encouragés et soutenus font aisément justice d'une telle appréciation.

Par les divers aspects de son activité, par une heureuse union de l'action et de la pensée, Clément Colson se présente à nous comme une des figures les plus robustes et les plus originales de notre temps. Son goût du réalisme allié à ses dons d'abstraction lui ont permis de faire œuvre durable dans le domaine de la science économique et d'aller plus avant dans les voies qu'avaient ouvertes les pionniers au cours du siècle dernier.

L'ayant eu pour maître, puis fréquenté plus intimement, nous ne saurions oublier ce que nous lui devons et nous nous félicitons d'avoir ici trouvé l'occasion d'apporter publiquement à sa mémoire le témoignage de notre affectueuse gratitude.

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PRICE AND WAGE POLICIES AND THE THEORY OF EMPLOYMENT

By Richard M. Bissell, Jr.

1

In spite of considerable work on the subject in recent years, it is fair to say that there is still a wide divergence of opinion in regard to the effects of wage policy and of wage-rate changes upon income and employment. Until rather recently economists have been agreed that reductions in wage rates would tend to increase employment whether carried out in particular industries or throughout the whole economic system. There has never been any agreement on the elasticity of demand for labor in general and it has always been recognized that wage changes will have different effects if carried out in different sectors of the economy. Nevertheless, it has been believed that the elasticity of demand for labor is positive even if, under certain circumstances, small.

It is always dangerous to attempt to state an economic theory and call it "the orthodox theory," especially when it is to be compared with other and divergent views. Nevertheless, a word about the proof advanced for the above proposition is in order. Of course, many have argued that, because a decline in wage rates will increase the volume of employment within a single firm or industry (assuming demand for its product to be unaffected) a general wage reduction would increase the demand for labor throughout the economic system. But it would be most unfair to suggest that any reputable economist has advanced this argument and, in so doing, ignored the repercussions of a wage change upon the demand for goods, i.e., upon consumer purchasing power. In the form in which it is usually advanced the argument is that a wage reduction involves (1) a change in the distribution of income within the community, and (2) some decline in the velocity of circulation, because profit payments are usually made only at the ends of accounting periods. It is conceded that both of these changes are likely, in some measure, to offset the favorable effects upon employment of the original wage reduction. But, since wages constitute a much larger proportion of prime costs than of total income, it is argued that the net effect of a reduction will still be favorable to employment. In other words, since a given wage reduction will constitute a much larger proportionate decline in marginal costs than in income, it is likely to give rise to an expansion of output and of employment unless prices throughout the system are highly inflexible. Another argument often advanced is that a wage reduction makes labor cheaper relatively to other factors of production, in particular, capital equip-

ment so that a wage reduction will induce some displacement of other factors by labor even if output remains unaltered. The latter argument, however, loses much of its force if there is any likelihood that wage reductions will be reflected in the prices of new capital goods. For, in that case, capital costs will decline nearly as much as wages.

The contrary view (that wage reductions will leave employment unaltered because they will involve so great a decline in consumer purchases) is at least as ancient as the so-called orthodox theory. It has been expressed many times in underconsumptionist writings of one sort or another. But it is only in recent years that it has achieved a considerable measure of respectability in the form in which it has been stated in Keynes' *General Theory* and in the work of Lerner, Harrod, and others. The proposition proved in the *General Theory* is that changes in wage rates will have no effect upon employment except via (1) the rate of investment and (2) the community's propensity to consume. Keynes points out that the change is likely to affect the rate of investment by altering both interest rates (given a certain type of monetary system and certain habits with respect to the holding of funds) and the marginal efficiency of capital. Also the probability that the change in wage rates will change the distribution of income and consequently the community's propensity to consume is taken into account. It is argued, nevertheless, that it is only through these channels that wage rates can affect employment.

There can be no doubt in the mind of anyone who has read through this work with care that the proof advanced is formally valid. Given the premises Keynes employs, his conclusions inevitably follow. Nevertheless, the whole thing has an air of being too easy and too abstract. Once one is satisfied that the reasoning is correct, one has an uneasy feeling that Mr. Keynes has produced a rabbit out of a hat and that he has managed to prove a most important proposition about the real world by the use of a few apparently innocuous assumptions. The Keynesian theory has been amplified by other writers, notably, A. P. Lerner[1] and E. M. Bernstein,[2] and their work has involved more explicit statement of certain of the assumptions and of the reasoning itself than appeared in the *General Theory*. The purpose of this paper is to examine a part of the still unresolved issues that have been raised by the development of a doctrine so sharply at variance with the early and more orthodox views referred to above. These issues are, roughly, of two sorts. First: those that concern the whole method of analysis

[1] "Wage Rates, Investment and Employment," *Journal of Political Economy*, Vol. 47, No. 2, April, 1939.

[2] "The Relation of Wage Policies and Price Policies," *American Economic Review, Supplement*, Vol. 39, No. 1, March, 1939.

employed in the *General Theory* and elsewhere, especially the use of so-called aggregative concepts. Second: those that concern the validity of certain specific assumptions made in the *General Theory* and the effect of changing a number of the assumptions. Perhaps the major thesis here advanced is that the method, if amended and generalized in certain respects, is highly useful, perhaps essential, for the analysis of wage and price problems and that this method, or one similar to it, must be employed for an investigation of specific changes in wages and prices within particular firms and industries as well as of general changes in the price and wage level.

2. SAVING AND INVESTMENT

It is scarcely necessary to devote much space to the definition of an elaborate set of concepts and to a discussion of the relationships among them. A few comments, however, on the conceptual apparatus are in order. It is now generally agreed (as pointed out by Haberler) that realized savings and realized investment may both be defined as the increment of total income not consumed in any accounting period, and that, when so defined, saving and investment are, of course, identically equal. This is possible and this relationship holds whatever definitions of income and of consumption may be employed, for the terms saving and investment, thus defined, are merely two ways of designating the same quantity. However, it is useful to bear in mind that it is only *realized* saving and investment that are identical. Both realized saving and realized investment (saving and investment *ex post*) differ from what Hawtrey calls active or planned saving and investment which is approximately equivalent to the saving and investment *ex ante* of the Swedish economist. There are certain difficulties about the exact definition of saving and investment as *ex ante* concepts. But the most important discrepancies between planned and realized saving and investment are those that arise in the course of any process of change. If a firm's or an individual's income is expanding or contracting, there is likely to be a lag between the actual changes in income and the adjustment of the rate of consumption to those changes. As a result situations will appear in which an individual's or a firm's rate of expenditure, still based on a previous level of income, is higher or lower than it will be after the adjustment to a change in income has been completed. It is the excess or deficiency of saving or investment which occurs under these circumstances that gives rise to a difference between planned and realized saving or investment. Thus, in general, planned saving and investment may be defined as that which would be carried on stably, given the current level of income.

For the purposes of the present discussion it is not necessary to say

much more about the definitions of income, saving, and investment, except to point out that, in order to be consistent with the above described usage of terms, an individual's or firm's saving must be defined as the excess of income over consumption, in the case of an individual, and over current cost disbursements, in the case of a firm. This definition always raises the ugly problem of the treatment to be accorded to depreciation and of the meaning of maintaining capital intact. It may be suggested that much the most practical useage to follow in this matter is to assume that the foreseen decline in the value of the capital owned by an enterprise is measured by accruals to its depreciation reserve. If this is done, accruals to depreciation reserves, together with other interest payments, must be subtracted from the firm's receipts in computing its net income. Now, the difference between a firm's net income and its actual disbursements to owners or stockholders is a measure of business saving. If accounting categories are employed, therefore, the volume of business saving is made to depend upon the size of annual accruals to depreciation reserves. This amounts to treating accruals to these reserves as if they were cost disbursements on a par with actual payments for labor, raw materials, and the like. Actually, however, such accruals do not measure real disbursements at all. It is, therefore, necessary to regard the excess of accruals to depreciation reserves over actual expenditures for replacement and renewals in any given accounting period as disinvestment and, conversely, to treat any excess of replacement and renewal expenditures over accruals to depreciation reserves as investment. By adopting this convention, the sum of consumption and investment measures the value of output actually produced with a deduction already made for the foreseen decline in the value of existing capital assets and, at the same time, the identical equality of investment and saving is preserved. These are, of course, definitions of net rather than gross saving and investment.

3. INCOME ANALYSIS

In any investigation of the effects of price and wage changes which is to transcend limits of partial-equilibrium theory, it is necessary to trace out the repercussions of an initial impulse upon economic decisions throughout the economy. Such an investigation must, therefore, focus attention upon relationships which are not treated in partial-equilibrium theory, in particular on the connection between the disbursements of certain individuals and enterprises and the incomes of others, and on the relationships in time between changes in the receipts of individuals and enterprises and the consequent changes in their disbursements, and vice versa. In addition, of course, it must

be concerned with the way in which enterprises' and individuals' disbursements for various purposes depend upon their receipts from various sources, which is the familiar subject matter of partial-equilibrium theory. Indeed, the variables that appear in all of these relationships are the ones treated in partial-equilibrium theory for they are incomes and expenditures.

The major difference between the type of analysis employed in an investigation which transcends partial-equilibrium theory and that type of theory itself is that the former, in contrast to the latter, is not concerned merely with the way in which an individual or firm adapts itself to a given situation but rather with the way in which its decisions, by altering its receipts or disbursements, impinge upon other members of the economic system. Since the actions of particular institutions or individuals are felt throughout the system by way of their influence upon the volume of money transactions, the broader investigation may conveniently be referred to as income analysis. It is, as this description suggests, a theory of the relationships among the more important streams of money income in the economy.

This characterization applies equally well to reasoning of the sort exemplified by the *General Theory* or to any set of propositions stated in the terms of the quantity theory. But this does not imply that all alternative sets of concepts are equally useful for the purposes of income analysis. Since income streams are the basic variables in terms of which the reasoning must be carried on, there is, for many purposes, an advantage in employing concepts all of which refer to time rates of receipt or disbursement of funds rather than concepts stated in terms of the quantity of money. In other words, there is, it may be suggested, an advantage in using what Marget has called an income theory rather than a quantity theory. The reason is that it is possible, using the income approach, to set up a conceptual model which will yield determinate results by making certain assumptions as to the forms of the relationships among income streams, which assumptions have economic significance. If the quantity-theory approach is used, however, it is difficult both to use assumptions which do have economic significance and to obtain determinate results. For instance, starting with the quantity theory as a general proposition, it is easy to conclude that a reduction in wage rates means a reduction in P and also, probably, in V and may, therefore, either increase or decrease T, but it is hard to get very much further. Of course, it would be perfectly possible to assume that a reduction in wage rates will increase the value of the fraction MV/P and will, therefore, increase employment. Nor would this procedure be any more arbitrary than that which Keynes employs in the *General Theory* to show that, under

certain assumptions, a reduction in wage rates will not lead to an increase in employment. The difference is, however, that the assumptions which Keynes makes, even if misleading, do have an economic significance which is far more easily perceived than is that of the assumption which might be made in terms of quantity-theory concepts. Because it is possible to state assumptions in the terms of the income approach which do have economic significance and which at the same time yield determinate results, it may be said that this type of analysis is somewhat better adapted than the other to quantitative investigation.

When it is said that the significant variables in income analysis measure the size of certain selected streams of money income in an economic system, the term income is used to refer simply to any series of money transactions. Thus a single stream of money income may be defined with reference to the receipts or disbursements of any one designated group of accounting units in the economic system from or to any other group. Clearly, the results achieved and the utility of the device will depend in considerable measure upon what are regarded as the significant groups.

What certain of them are is readily apparent. The primary division usually made is that between firms or entrepreneurs on the one hand and, on the other, individuals regarded as receivers of income and consumers. Often the latter group is broken down into wage earners and others, a separation which is of particular importance, obviously, for a discussion of wage changes. In the Keynesian analysis this matter is not explicitly discussed but, by implication, the owners of enterprises are associated with the firms themselves in the same group and the only individuals referred to, other than wage earners, are receivers of fixed income.

It greatly increases the utility of income analysis, however, if the designation of groups is carried somewhat further. In the first place, we may define a category of firms and individuals other than wage earners who supply (primarily to business firms) certain services or commodities, or even the use of such factors of production as land. Such units may be designated as suppliers. In the second place, we may separate out explicitly the category of owners as a separate group. It should include receivers of fixed-interest incomes as well as owners of common stock, but it is probably wise to exclude from the category of owners small independent entrepreneurs and to treat them, rather, as suppliers. The category of suppliers should also be taken to include individuals who render professional services and the like.

The reason for setting up such categories as these may not be immediately obvious and, indeed, the procedure is perfectly arbitrary.

Positively stated, however, its purpose is to separate out, however roughly, types of economic units according to the way their incomes and disbursements behave. For instance, if all entrepreneurs and owners of common stock are associated with large enterprises in a single group, no one assumption about the way in which disbursements and incomes are related is at all realistic. A change in the net income earned by the United States Steel Corporation is simply not the same thing as a change in the personal incomes of its common-stock holders. If steel production goes up, it does not follow that the purchases of consumers' goods by common-stock holders will immediately respond. On the other hand, this identification of the enterprise with a consuming unit is undoubtedly valid in the case of most farmers and other small entrepreneurs. On many grounds, indeed, the latter could be identified with wage earners so far as their position in the economic system is concerned. An increase in output will usually lead to the employment of more wage earners and to larger purchases from small entrepreneurs. The latter may be expected to react to changes in their incomes in exactly the same way in which the former do. In the light of such considerations as these, then, it will add to the flexibility of income analysis if both small entrepreneurs and owners of common stock are kept separate from large enterprises in any grouping of accounting units.

The desirability of treating stockholders and rentiers in the same category is, of course, open to doubt. But the arguments in favor of so doing are familiar. The owners of small enterprises, even if incorporated, should probably be treated as suppliers. Their incomes are likely to vary with much the same lag as those of small entrepreneurs. But in the case of large corporations, the disbursements to common-stock holders are apt to be a good deal more stable than most types of cost payments. It has been suggested (by E. M. Bernstein) that corporate disbursements to stockholders show a close correlation with their disbursements to wage earners. If this is true, then common-stock owners could be grouped with wage earners in the same category. In any case, the main point is that common-stock holders should not be associated in the same group with the enterprises which they own.

A. P. Lerner has recently suggested another modification of the Keynesian picture. He discusses the effect of wage changes under the assumption that there exists a second "original" factor of production on a par with labor which could be land or another type of labor. It is, of course, essential that income analysis should be sufficiently flexible to take account of this case. In the terms developed above, landowners could be included either in the category of suppliers or in that of owners. If their incomes are conceived of as "fixed" incomes, i.e., if it is assumed that they are determined by contract and do not vary with

fluctuations in income except after a substantial time lag as is the case with bondholders and many common-stock holders, then they should be included in the latter category. Lerner seems to have in mind a factor of production which may have a fixed price but of which larger or smaller amounts are employed as production increases or diminishes. Such a factor should, of course, be included in the category of suppliers.

To sum up the above discussion: the groups which have to be separated out for the purposes of income analysis and in terms of which it can be carried on are firms, wage earners, owners, and suppliers. Others could be added to this list but there is a real danger that the analysis will be misleading unless at least four such groups are employed. What is rather more important is the principle upon which they should be distinguished. Since income analysis deals with streams of income, it is desirable to define categories with reference to (a) the rapidity with which their incomes are affected by changes in aggregate income and in the level of production, and (b) the way in which their disbursements react to changes in their receipts. Rentiers must be separated out from other categories in the community because their incomes do not fluctuate nearly so promptly with fluctuations in aggregate income. Small entrepreneurs must be separated out from large corporations because their expenditures fluctuate much more promptly in response to changes in their net incomes. Common-stock holders must be separated out from the enterprises which they own on both grounds.

One last comment must be made on this scheme. It should be obvious that we are here discussing categories of income rather than categories of people. Most common-stock holders are also rentiers, many of them are at the same time wage earners, and they may be suppliers as well. This fact may have some significance in connection with the assumptions that are usually made about the habits of firms and individuals with regard to the disposition of their incomes. It seems very unlikely, however, that the validity of assumptions or of the reasoning in which they are employed will be in any way affected.

Once it is decided what grouping of incomes or of economic individuals is to be employed, it is possible at once to pick out the important income streams through which cause and effect relationships are to be traced. Following the pattern of most economic theory, income analysis is concerned predominantly with the decisions of business enterprises. Beginning, then, with income streams defined with reference to firms, their disbursements will consist of payments to wage earners, owners, suppliers, and other firms. The aggregate disbursements of all firms taken together will include only payments to wage earners, owners, and suppliers. Transactions between firms cancel out

and do not appear in aggregate disbursements or aggregate receipts.

The fact that disbursements can be distinguished according to the group to which they are made suggests that the same principle should be employed in a classification of receipts. But for various reasons, it is useful to divide up receipts on the basis not merely of the groups from which they come but also of the types of expenditure which give rise to them. Specifically, they must be distinguished according as they grow out of consumption or investment expenditures. The reason is that, as a rule, individuals use substantial portions of their income to purchase securities or to increase their cash balances rather than for the purchase of goods and services. In consequence, the aggregate disbursements of firms to individuals, (owners, suppliers, and wage earners) are regularly less than their aggregate receipts from them. The funds thus saved by individuals do not reappear as business receipts until they have been secured by business firms through capital transactions and spent to finance investment. It would, therefore, be quite impossible to reach any conclusions about business saving and profits and about the incentives to produce larger or smaller outputs of goods if attention were limited to the aggregate receipts of business firms from individuals. Instead, it is convenient to include in aggregate receipts not only the sum of owners', suppliers', and wage earners' consumption, but investment as well. This may at first appear to be a curious usage of the term because investment is usually defined as a certain type of expenditure. But the only assumption involved in including it among the aggregate receipts of firms is that investment expenditures are payments to enterprises rather than directly to individuals in the first instance and this is, surely, not too unrealistic or damaging to the generality of the analysis. The practice amounts to including in aggregate receipts this one element of receipts from other business firms as well as the actual current receipts of firms from individuals. The class of aggregate business receipts, then, includes all receipts of firms from members of other groups and a designated part of the receipts from other members of the same group. It is clear that gross receipts, thus defined, do not measure the national income in any sense for they exclude the value of output produced and services rendered by any productive units which are not included in the group of business firms. This difficulty could be got around by regarding any producer as a "firm" but, as suggested above, this is not a useful procedure. A second point regarding the suggested usage is that, in many cases, investment carried on by a firm during an accounting period does not represent entirely or, perhaps, even largely, disbursements to other firms but, instead, the costs of construction, repairs, and improvements carried out by the firm itself. When this is the case, invest-

ment will have all of its usual consequences but it will not measure actual receipts by one business unit from another.

Once aggregate receipts and disbursements are defined, business saving may be described simply as any excess of receipts over the sum of current disbursements and accruals to depreciation reserves, where current disbursements are all except expenditures for renewals and replacements. In considering the way in which any given event will change aggregate business saving, however, it is sufficient to study its effect upon the excess of aggregate receipts over aggregate disbursements. For any increase in this latter quantity must be the result of an increase in receipts, a reduction in current disbursements, or a reduction in expenditures for replacement and maintenance. An increase in receipts or a reduction in current disbursements (with no change in accruals to reserves) obviously constitutes an increase in business saving. A reduction in maintenance or replacement expenditures unaccompanied by a change in accruals to reserves constitutes either a reduction in the rate of investment or an increase in net disinvestment. Thus, any change in the excess of aggregate receipts over aggregate disbursements must measure a change in business saving in one direction or in investment in the other, and the way it is treated is immaterial.

The aggregate receipts of any one of the other groups will include business disbursements to its members and receipts from other individuals. Likewise, the aggregate disbursements of one of the other groups will consist of consumption, disbursements to other individuals, and the excess which, by definition, constitutes saving. Throughout most of the analysis which follows, however, transactions between different groups of individuals, say, wage earners and suppliers, will be disregarded.

4. ASSUMPTIONS

Because of the very nature of income analysis, the most important of the assumptions upon which it is based are those which must be made about (a) the relationship between changes in disbursements and in receipts, and (b) their timing. These two types of assumption are by no means clearly distinguishable. The easier ones are those that concern individuals. First: as to the disposition of their incomes, the usual assumption is that the proportion of income saved is an increasing function of the size of real income. Another way of stating this is that any given change in an individual's income will lead to a less than proportionate change in his consumption. One comment should be made on this assumption. In the *General Theory*, Keynes chooses to measure such quantities as consumption and income in terms of wage

units. This is equivalent to measuring them in money with a correction for changes in the level of wages. If this mode of definition is strictly adhered to, it raises quite preposterous difficulties unless prices are assumed to be highly flexible. Under the assumptions of the *General Theory*, it seemed reasonable to take the wage unit as a measure of the unit of real income and it would appear to be following the spirit if not the letter of Keynes' assumption to state the above proposition in terms of real income instead of income in wage units.

This takes care of the size and direction of the change in consumption which follows the change in income but not of its timing. No general assumption is required, however, as to the time lag between income and disbursement changes except that it should not be so great as entirely to interrupt the process of transition set in motion by any change in incomes. Clearly, this assumption is not at all precise but it need not be so for it is not the absolute length of these time lags that is significant, only their length as compared with that of certain others. Whatever the relative length of time lags, it is always true that the shorter they are the sooner any transition will be completed, and conversely.

Turning to the receipts and disbursements of firms, rather more complex considerations must be taken into account. Not only are there different types of disbursements which are related to the volume of receipts in different ways, but, in addition, the relationship between changes in receipts and changes in disbursements will depend upon the manner in which or the purpose for which the initial change is made. As the first step in developing a realistic assumption about the nature of these relationships, it may be well to point out that disbursements to wage earners, suppliers, and other firms are of somewhat different character from those to owners and rentiers in that they are neither fixed by contract nor residual shares. These will be referred to as cost disbursements. The aggregate cost disbursements of business firms include, however, only those to wage earners and suppliers. It is with these that we shall, for the moment, be mainly concerned.

What chiefly complicates the relationship between receipts and cost disbursements is that changes in these quantities may or may not be accompanied by changes in output. If receipts are altered as the result of a change in demand unaccompanied by a change in price, then there will be a change in output, but, if it is possible, as the result of a change in demand, for a firm to alter its price without altering its output, the effect on its disbursements will be entirely different. The most general case is that in which there is a change in price which has an effect upon output so that there are two components to the change in receipts. Finally, it should be pointed out that there may similarly be changes in

selling costs which either give rise to changes in output or else merely counteract changes in price or in demand conditions, so that they are not accompanied by changes in output. The size and nature of the related changes in receipts will depend upon the nature of the original variation in selling costs. In view of these complexities it will clarify the discussion if a price reduction or an increase in outlay to finance a product variation or heavier selling costs is conceived of as having two separate effects, the sum of which is the net effect upon the firm's position. The first is the effect which the price change or increase in outlay would have if it were not accompanied by any change in output. The second is the change in output that would have to be added to the first effect in order to give the total effect. For instance, the total effect of a price reduction will be to lower average receipts but to increase volume, so it may increase or reduce the firm's gross revenue. But this may be treated as the sum of (1) the price reduction unaccompanied by the increase in output, and (2) the expansion in output at the new lower price. Similarly, an increase in, say, advertising outlay will, other things being equal, bring about an increase in the firm's sales and gross revenue. In this case the total change may be regarded as the sum of (1) the increase in outlay, and (2) the resulting increase in sales and in revenue. In both cases, the first effect will be either to reduce the firm's receipts or to increase its cost disbursements and the second will involve an increase in its revenue. Only if the second change outweighs the first will the total effect be favorable to profits.

This distinction facilitates the making of an assumption about the relationship between changes in a firm's receipts and changes in its disbursements. It is fairly safe to assume that any change in output will alter cost disbursements to both wage earners and suppliers and to assume further that the time lag between the change in output and the change in cost disbursements will be exceedingly small. (This will be true, of course, of at least many types of disbursements to other enterprises as well but these do not enter into aggregate cost disbursements.) Since the connection between output and cost disbursements is clear, we may say that any increase in receipts which results from a spontaneous increase in demand or which comes about as the second effect of an increase in selling outlays or of a price reduction must be accompanied promptly by an increase in cost disbursements. It is equally clear that the change in revenue which comes about as the first effect of a price change will not have any effect upon cost disbursements because, by hypothesis, it is not connected with a change in the rate of production.

A reasonably safe assumption may also be made as to the relative

magnitudes of changes in cost disbursements and associated changes in receipts if it is remembered that associated changes in receipts exclude the first effects of changes in prices and selling costs, for this makes it possible to treat every change in output as if it were the result of a change in demand and to take account of the first effects of price and selling-cost changes separately. Unless a firm has in some way established a price which is below marginal cost, any increase in its output is bound to lead to an increase in variable costs which is smaller than the associated increase in receipts. In general, the change in receipts is bound to be greater than the associated change in cost disbursements. If it is desirable to describe all of the effects of, say, a price reduction, it may be said to have a first effect upon receipts which will be negative and will not be accompanied by any change in disbursements and that it will have (perhaps simultaneously) a second positive effect upon receipts which will be accompanied by a somewhat smaller positive effect upon cost disbursements. As this implies, there is no necessary priority in time of the first effect over the second.

The above assumptions cover the relationship between, on the one hand, initial changes in business receipts and certain sorts of initial changes in business disbursements and, on the other hand, resulting changes in cost disbursements, given the prices of the different types of input employed by business firms. Problems somewhat analogous to those just discussed arise, however, when the initiating impulse is a change in the price of input. If attention is confined to aggregate receipts and disbursements, then the two types of price change that can have an immediate impact upon costs are changes in wage rates or in the prices of the services of the suppliers or products produced by them. In both cases, it would be possible, following the method outlined above, to distinguish the first effect upon business receipts and disbursements and upon those of the other groups involved and a separate second effect. The first effect is that which would occur before there has been any change in the employment of labor or of the services or products of suppliers. The second effect would be that which would result from the change in employment of services or products at the new prices. Since the two groups of suppliers and wage earners are assumed to stand in similar positions in the community, that is, to consist of individuals selling services or products to business firms and receiving personal incomes in payment, any income analysis should be completely symmetrical with respect to both groups. The results of a change in the prices of suppliers' products or services could be followed through in precisely the same way as the results of a wage change. Nevertheless, in this paper, attention will be confined to the results of wage changes, i.e., the prices of suppliers' services or products

will be assumed to be constant. This being the case, there will be no changes in disbursements to or receipts of suppliers which can be called first effects of price changes. Disbursements to and incomes of suppliers will change only as the result of changes in output and in the amounts of such goods or services purchased as input by firms.

Up to this point nothing has been said about firms' disbursement. to owners which are not among other cost disbursements. Since small entrepreneurs and even very small business enterprises must be included in the group of suppliers and since the disbursements of firms to owners include, therefore, only disbursements by or withdrawals from enterprises of substantial size, it is almost certainly reasonable to assume that changes in these disbursements will follow changes in business receipts (if these are of such a sort as to give rise to changes in the net incomes of business firms) with a much greater lag than changes in cost disbursements. There is, furthermore, some reason to to believe that this lag between changes in business earnings and in their disbursements to owners is much greater than that between changes in the incomes and expenditures of individuals whether wage earners, suppliers, or owners themselves. If this is true, then it is conceivable that an initial change in wages or prices might give rise to certain changes in employment and in incomes which would work themselves out fully and would lead to a new position of stability before any change had occurred in business disbursements to owners. This is clearly an extreme and limiting case, in itself unrealistic. Nevertheless, if, as a working hypothesis, it is assumed that, in the short run, disbursements to owners are constant, the significance of the time lag between changes in earnings and changes in disbursements will be brought out. The hypothesis may then be relaxed and the more realistic possibilities explored. It is worthy of emphasis for the difference between the results obtained in the first cases treated in this paper and the results obtained by Keynes in the *General Theory* are due entirely to its use. Keynes, as pointed out above, treats the owners of the enterprises and the enterprises themselves as identical, so there is no place in his reasoning for the time lag between changes in earnings and changes in personal incomes derived from profits.

It has been necessary to discuss in considerable detail assumptions about the relationships between receipts and disbursements and the timing of changes in these quantities because these are the less familiar and the more important assumptions of income analysis. But something must be assumed, too, concerning the way in which the rate of investment is determined, the monetary system of the community, and the whole determination of the rate of interest. Any thorough considera-

tion of wage and price changes must take account of the possibility that they will influence both the incentive to invest and the rate of interest. It will simplify the analysis greatly, however, if we assume provisionally that the rate of interest and the marginal efficiency of capital can be taken as given, so that the rate of investment also becomes a constant.

Finally, and most important of all, some working hypothesis must be made as to the relationship between prices and wages. The assumption made in the *General Theory* is that prices are determined by individual producers in such a way as to equate marginal revenue and marginal costs. This implies that the difference between prices and marginal costs depends upon elasticities of demand. Furthermore, it is assumed that elasticities of demand are not greatly altered by changes in wage rates so that prices change by approximately the same proportion as marginal costs when wage rates are altered. When only aggregate quantities are under consideration it is reasonable to assume that wage costs constitute a large proportion of aggregate variable costs and, therefore, of aggregate marginal costs. Keynes finally emerges with the assumption, therefore, that prices remain constant in terms of wage units which is equivalent to assuming that they change by the same proportion as wage rates in the short run.

This is obviously not the only assumption that can be made. For one thing, prices could be completely inflexible so that they would be unaffected by changes in the wage rates. But, even supposing prices to be, in the normal sense of the word, perfectly flexible, it is not true that they will vary by the same proportion as wage rates. This is for two reasons. First: if purchases from suppliers constitute a substantial part of aggregate variable costs, then marginal costs will not vary proportionately with wage rates, even in the case of general wage changes throughout the system. Under these circumstances it is reasonable to assume that prices will vary proportionately with marginal costs and, therefore, by a smaller proportion than wage rates. Second: if marginal revenue is equated to marginal costs in equilibrium and if it is substantially less than price, then it is possible that prices will vary by the same *amount* as marginal costs rather than in the same proportion. The difference between marginal revenue and price depends solely upon the elasticity of demand. If elasticity remains unchanged when costs and prices are altered, there does not appear to be any reason why the margin between marginal costs and price should be narrowed when both move downward or widen when both are raised. If this is true, then, except under perfect competition prices will move by the same amount as marginal costs but by a somewhat smaller

percentage. To be more precise, the percentage change in price will be equal to the percentage change in marginal costs multiplied by the ratio of marginal costs to price.

The first of these two considerations is of universal applicability and we shall, therefore, assume as the first type of price flexibility that prices move proportionately with marginal costs which means by a smaller percentage than wage rates. It is for several reasons useful to treat this first type of price flexibility and ignore the effect of differences between marginal revenue and price. One is that it is a close approximation to the actual situation whenever the degree of monopoly is small. Another is that firms may frequently have an incentive to move prices up more than the actual amount of the change in marginal costs when wage changes compel them or permit them to make some change. In any event, the limiting case is that in which prices change proportionately with marginal costs. It will not be difficult after this has been developed to indicate how it would have to be modified if we assume what may be called the second type of price flexibility in which prices move by a slightly smaller percentage than marginal costs but by the same amount.

One last assumption must be made in order to simplify the discussion. In view of the fact that wage costs and disbursements to suppliers comprise the whole of aggregate variable costs, it is a reasonable presumption that changes in wage rates will alter *average* variable costs and *marginal* costs by the same percentage. The assumption is useful because it permits us to reason that a change in wage rates will, in the absence of changes in output, have the same proportionate effect upon disbursements to wage earners and suppliers jointly as upon marginal costs.

Although no further assumptions are required, a word must be said as to the procedure to be adopted in examining the effects of wage changes. For many analytic purposes it is useful to split the total effect of a wage or price change into two parts and to speak of, on the one hand, the income and, on the other, the substitution effects of the change. This distinction was originally developed to analyze the results of price changes in particular markets or the effects of price changes upon particular individuals. In this context, the income effect is that which results from the fact that a price change alters real incomes and the substitution effect that which results from the fact that an absolute price change necessarily constitutes a relative price change as well. When these two terms are employed in connection with income analysis, their meanings cannot be given quite so easily but the essential nature of the distinction is clear. A price or wage change, whether in a particular market or throughout the whole economic system, will

have one set of effects because it will alter the real incomes of certain firms and individuals via money receipts and of others via price changes. At the same time, it may itself constitute a relative price change or it may give rise to such changes. Insofar as it does so, its final results will depend upon the extent and the nature of the substitution of goods for goods, or of factors for factors that it causes. The first set of effects may be designated income effects; the second, substitution effects.

Much income analysis recently developed to deal with the problems of general price and wage changes is concerned almost exclusively with income effects. On the other hand, there has been a tendency for orthodox analysis based on the quantity theory to emphasize substitution effects. This, incidentally, is one of the most important sources of the disagreement noted at the beginning of this article. Once the difference has been described in these terms, there can be no question of one approach or the other being correct, for it is only by summing the two types of effects that a complete analysis can be made. Nevertheless, the task of constructing a model is rendered far simpler if the two types of effects are considered separately. This procedure will be followed here and in the immediately following sections attention will be devoted exclusively to the income effects of wage changes. Thereafter it will be possible to give explicit consideration to the nature of substitution effects and to see how they exaggerate or modify the income effects.

5. GENERAL WAGE CHANGE, PRICES FLEXIBLE

Any complete analysis of the effects of wage changes must, of course, take into account the various types of price behavior that might be encountered between the two extremes of flexibility and rigidity as well as the two limiting cases themselves. Nevertheless, the limiting cases are, as always, the most clearly defined. We may, therefore, begin with the first type of price flexibility. Let us assume that wages are reduced by, say, ten per cent in money terms. Marginal costs will be reduced by somewhat less, say, eight per cent, because nonwage variable costs will not have been affected. Under the assumption of the first type of price flexibility, prices will also be reduced by eight per cent unless and until there are changes in output that have the effect of raising marginal costs. Given our assumptions, it is possible to list certain components of the result of the wage reduction and to group them as follows:

(1) There will be an effect of the first type, that is, one that would follow even if output remained unaltered. Wage earners' real incomes will be affected adversely, because prices are cut less than wage rates. On the other hand, suppliers' real incomes are substantially increased

by the reduction in prices, in view of the fact that prices will be reduced by approximately the same percentage as the disbursements to wage earners and suppliers taken together. This implies that the real income of the two groups treated jointly will not be altered in the absence of a change in output. Disregarding the differences between their propensities to consume, it follows that their saving and consumption in terms of dollars will both be cut by the same proportion by which prices and their aggregate money income have been reduced. Thus, the absolute cut in disbursements by firms to wage earners will be largely but not wholly offset by a proportionate cut in the joint consumption of wage earners and suppliers. What will actually happen, of course, is that wage earners' consumption will be cut by a smaller percentage than their money incomes because of the reduction in their real incomes and consequent higher proportion of income spent. But suppliers' consumption, measured in dollars, will decline slightly because of the increase in their real incomes and consequent increase in the proportion of income saved. For the reasons just given, however, the net effect is likely to be a cut in joint consumption approximately proportional to the cut in joint income.

(2) Investment receipts will be reduced by the same proportion as prices. This follows from the assumption that the real volume of investment remains the same and that prices of all types of goods are flexible.

(3) There will also be an effect of the second type on wage earners' and suppliers' incomes, that is, one that results from a change in output. As disbursements to owners are assumed to be initially unchanged, their real incomes will be increased by price reductions. Under the usual assumption, this will lead to some reduction in the dollar volume of owners' consumption. But, as the real consumption of owners is certain to be substantially increased, one of the effects of the wage reduction must be an expansion of output with its accompanying increases in disbursements to suppliers and wage earners.

It is clear that the expansion in disbursements to suppliers and wage earners which comes about as the second effect (in the sense in which this term was used above) of the wage and price reduction, that is, as the result of the increase in output, will offset partly or wholly the reduction in disbursements to wage earners which would be the effect of the wage cut without a change in output. Furthermore, the expansion of output and this second effect upon wage earners' aggregate income will have a series of further repercussions. These I shall refer to as the induced effects of the increase in output. They must be added to those effects of the wage and price cut listed above because we have already taken account of the way in which the first effect of the wage cut on

wage earners' incomes will affect their consumption and we have listed this cut in their consumption as one result of the wage and price change. The reason for thus treating the first and second effects upon wage earners' and suppliers' incomes separately and later adding them together to discover the net effect is simply one of exposition. It is not intended to suggest that there is necessarily any distinction in time between them.

Before taking account of the further effects of the expansion in output, i.e., the induced effects of the original wage and price change, it may be well to sum up the results so far listed. There will be a first reduction in disbursements to wage earners, largely offset by a proportionate decline in the dollar consumption of wage earners and suppliers considered jointly, a decline in investment receipts proportionate to the decline in prices, some reduction in owners' dollar consumption, and an increase in cost outlays sufficient to finance the increase in production required by the larger real consumption of owners. If owners did not increase their saving as a result of the rise in their real incomes, there would have to be an increase in output which would have the value of, in this case, eight per cent of their consumption. The net effect of these changes is bound to be unfavorable to business saving.

The induced changes are much easier to describe. The initial increase in cost disbursements required to finance a larger output has been included as one of the immediate effects. Since these increased disbursements constitute increases in the incomes of wage earners and suppliers, they will give rise to an induced increase in wage earners' and suppliers' consumption. This, in turn, will necessitate further expansions of production, employment, and disbursements to the same groups. All of these successive increases in employment and income constitute the induced effects. As the expansion continues, the planned saving of wage earners, suppliers, and of the business firms affected will increase. The expansion will continue until the induced increase in planned saving is equal to the initial increase in cost disbursements which was one of the direct or immediate effects of the wage and price reduction. The extent to which income and employment are increased in the process depends upon the marginal propensity to consume of the individuals whose incomes are enlarged and upon the rapidity with which business profits and, therefore, business savings grow as output expands. The relationship between these two factors determines, also, whether most of the induced increase in saving is individual saving or business saving.

It may serve to clarify the nature of the change here described and of the final position reached if the results of the process are described

more explicitly in terms of saving and investment. It follows from the general assumptions made that the dollar volume of investment is reduced by the amount by which prices are lowered. Since planned saving must be equal to planned investment before a new position of stability is reached, it follows that the total dollar volume of planned saving must be reduced by the same amount.

One of the immediate effects of the wage and price reduction is to reduce the saving (measured in dollars) of wage earners and suppliers jointly by the same proportion by which prices and the dollar volume of investment are reduced. But the increase in the real incomes of nonwage earners has the immediate effect of increasing the amount of saving which they perform. Furthermore, the increase in their real consumption gives rise to an increase in cost disbursements and, with it, an induced increase in private and business saving. One of the direct effects of the wage and price change, therefore, must be to reduce business saving by an amount large enough to offset first, the increase in owners' saving, second, induced increases in saving, and third, that portion of the decline in investment not compensated by the decline in wage earners' and suppliers' saving.

The second of these two amounts (the induced increase in saving) is equal, as just pointed out, to the cost of the increase in owners' real consumption. Thus, the three amounts may be described as (1) the increase in owners' saving, (2) the cost of the increase in their real consumption, and (3) the decline in that proportion of investment financed by owners' saving. For instance, if prices have been cut by eight per cent and if owners' saving does not increase (in spite of the increase in their real incomes), then the immediate effect upon business saving of the change in wage rates would be to reduce it by an amount equal to eight per cent of the investment financed by owners' saving plus the cost of an eight per cent increase in owners' real consumption. In the final position business saving would not have been reduced by this whole amount because some of the induced increase in saving would be business saving. Nevertheless, it must necessarily have suffered a very substantial reduction.

These results may appear somewhat paradoxical inasmuch as it turns out that a cut in wages has the effect of reducing business profits. But the paradox resolves itself once it is understood that what is really involved in a wage and price reduction when prices are flexible is an increase in the real incomes of owners together with a redistribution of real income as between wage earners and suppliers in favor of the latter. Indeed, the whole development could be stated more simply in real terms. If (as assumed heretofore) the shift of real income from wage earners to suppliers does not involve any change in their joint

saving or consumption in real terms, the wage and price reduction constitutes simply an eight-per-cent increase in the real income disbursed to owners. The increment may be divided up into three parts. First: the increase in real saving that comes about by reason of the reduction in the prices of capital goods. Second: the increase in real saving reflected in a different disposition of income by owners after their incomes have been increased. Third: the cost of the increase in their real consumption. But these three parts correspond to the three described above in money terms. If the whole change is thought of as an increase in real income disbursed to owners, it is clear that, in the final position, output and employment will have increased and there will be some induced business saving to offset part of the initial increase in real income disbursed to owners. The increase in the real incomes of owners, it should be noted, does not come about at the expense either of wage earners or of suppliers but rather at that of business firms themselves. It is not unnatural, therefore, that the final effect upon business saving should be negative and should approximate the value of the increase in the real incomes of owners.

It scarcely need be added that the shift of real income in favor of suppliers at the expense of wage earners might involve changes in the propensity to consume which would affect the position finally reached. For instance, to revert to money terms, if suppliers had a lower marginal propensity to consume than wage earners, the decline in their consumption measured in dollars which would result from the increase in their real incomes together with the decline in wage earners' consumption following upon the reduction in their real incomes might constitute a decline in the joint consumption of the two groups more than proportionate to the reduction in prices. If it did so, there would be a negative influence upon output which would offset in some measure the immediate increase.

The development following a change in wage rates will be rather different if price flexibility of the second type prevails. Under this assumption prices will be reduced by a smaller percentage than marginal costs as a result of a wage reduction. The actual price reduction for each individual commodity will be the same as the *absolute amount* of the reduction in marginal costs. Now it has been assumed throughout that marginal costs and average variable costs change by the same percentage when wage rates are altered. Therefore, wherever marginal costs are equal to average variable costs, price (average receipts) will be reduced by the same absolute amount as average variable costs after a wage reduction. Wherever marginal costs are greater than average variable costs, price will be reduced by a larger absolute amount than the latter, and where marginal costs are less than average variable

costs, price will be reduced by a smaller amount. Let us consider the special case in which marginal costs are everywhere equal to average variable costs. If, in this situation, wage and price reductions were to leave output unaltered, the decline in firms' receipts brought about by the changes in prices would be exactly equal to the reduction in total variable costs, that is, in the disbursements of firms to wage earners. We know that a part of the decline in receipts will represent a reduction in the dollar volume of investment because we have assumed that the real rate of investment is constant. The remainder must represent a decline in the dollar volume of consumption. What the above conclusion means, therefore, is that, if the reduction in disbursements to wage earners does not give rise to any change in the *proportion* of income that is spent in the whole community, then it will cause just that reduction in the dollar volume of consumption required to maintain the output of consumers' goods, i.e., real consumption, the same. Likewise, the reduction in saving that it will cause will just offset the reduction in the dollar volume of investment that will follow the decline in prices. To put the matter another way, what is required if output is to remain the same is that the ratio of the reduction in wage earners' consumption to the reduction in their incomes is the same as the ratio of consumption to incomes for the community as a whole. In other words, the marginal propensity to consume of wage earners must be the same as the average propensity to consume of the whole community. Given price flexibility of the second type and given these very restrictive assumptions, a wage reduction will reduce wage earners' real consumption but the reduction will be exactly offset by the increase in the real consumption of others.

It hardly need be pointed out that this outcome is most unlikely. But it does enable us to say that, in the special case in which marginal and average variable costs are everywhere equal, any change in real income and employment which results from a change in wage rates will be the result of a shift in the distribution of income and, consequently, in the propensity to consume. The reasoning can be followed through in exactly the same way when the assumption is relaxed. If marginal costs are greater than average variable costs, then, the reduction in price will be greater than that in average variable costs which means that disbursements to wage earners will be reduced by less than the reduction that would occur in the receipts of firms as a result merely of the price change itself. In this case, if wage earners have the same propensity to consume as members of other groups, the reduction in their consumption (in dollars) will be less than the reduction in their dollar volume of total consumption which would be required in order to keep real consumption the same and the reduction

in their saving less than the reduction in the dollar volume of investment required to keep real investment the same. Therefore, there would be some increase in employment. The case in which marginal costs are less than average variable costs is just the reverse.

6. GENERAL WAGE CHANGES, PRICES INFLEXIBLE

These conclusions have all been reached on the assumption that prices are flexible in one of the senses defined above. It will serve to throw them into relief and to indicate which assumptions are important and which are of little importance for the analysis if we turn now to the case at the opposite extreme and assume that prices are entirely inflexible, that is, that they do not change at all as a result of changes in wages and in marginal costs. It may be said parenthetically that this case is probably just about as close to reality as that of flexible prices.

Let us suppose, again, that there is a wage reduction of ten per cent. The immediate consequence of the change will be, of course, to cause a ten-per-cent reduction in the incomes of wage earners. Their consumption will be reduced by a somewhat smaller amount and thus the absolute reduction in their consumption will be less than that in their incomes. Since this represents a decline in wage earners' real incomes, a decline in production will follow and this will permit business firms to reduce still further their disbursements to wage earners and to effect a reduction in those to suppliers. (The disbursements to owners are assumed to remain constant as will their consumption and investment receipts.)

In addition to these immediate changes in disbursements and in receipts, there will be a further induced contraction in output, employment, and incomes. As the contraction proceeds there will be an induced decline in business saving together with a contraction of the volume of saving by wage earners and suppliers. The induced contraction will cease when the induced reduction in planned saving is equal to the cut in cost disbursements made possible by the original contraction in output.

As in the previous case, a mere recitation of these changes is no picture of what has occurred. Leaving aside induced changes, there are three elements which affect profits: (1) the first reduction in disbursements to wage earners, (2) the consequent reduction in receipts from them, and (3) the further reduction in disbursements to wage earners and suppliers made possible by the decline in production. The second of these will be smaller than the first to the extent of the cut in wage earners' saving. Thus, the increase in business saving that would result from these three changes unaccompanied by any change in out-

put is the sum of the drop in wage earners' saving and the cut in production costs that results from their lowered consumption. This sum is roughly equivalent to the value of the cut in their real incomes. Of course, the induced contraction will reduce wage earners' and suppliers' incomes further and it will also bring about reductions in profit so that, in the final situation, profits would not have increased by the full amount of the immediate cut in wage earners' real incomes. The decline in receipts from wage earners will be less than the first reduction in disbursements to them to the extent of the cut in their saving. Thus the immediate increase in business profits is the sum of the drop in wage earners' saving and the cut in costs resulting from their lowered consumption. This sum is roughly equivalent to the value of the cut in their real incomes. Of course, in the final situation, profits will not have increased by this amount because induced reductions in profit would accompany the induced contraction of output.

7. THE KEYNESIAN CASE

The case developed by Keynes in the *General Theory* corresponds to neither of the ones here developed and before going further it may be well to indicate how Keynes reaches the conclusion that a change in wage rates will leave the volume of employment unaffected. It depends upon one crucial assumption which is that business firms' disbursements to owners vary immediately with changes in business profits. This amounts to identifying business firms with their owners, as pointed out above.

Applied to a situation in which the first type of price flexibility obtains, it would produce exactly the same results that Mr. Keynes arrived at in the *General Theory*. As soon as the increase in the real incomes of owners, following wage and price reductions, made it necessary for firms to finance a larger volume of output and as soon as investment receipts (in dollars) declined business firms would promptly reduce their disbursements to owners. But this reduction in disbursements to owners would immediately reduce their real incomes to their original level, curtail their consumption and make the expansion of output unnecessary. It has been pointed out above that the immediate effect of the wage reduction would probably be to increase suppliers' real incomes by approximately the same amount by which it diminished those of wage earners. If this were the case, and if saving by these two groups had not altered, then, in the final situation, the real incomes of owners would also be unaltered. This is precisely the result reached in the *General Theory*. There could be a change in income and employment only if the redistribution of income were to change the community's propensity to consume.

It is interesting to see what happens in the case of inflexible prices if Keynes' method of analysis is employed. If firms were unwilling to increase their volume of saving except as a result of a substantial increase of output and income, then, so soon as a wage reduction with rigid prices showed any tendency to increase their profits, they would be under pressure to increase disbursements to owners. There would, in consequence, be an increase in owners' consumption which would tend to offset the decrease in wage earners' consumption. If, by any chance, the marginal propensity to consume by the two groups were exactly the same, then, in the final position, income and employment would remain unaltered. There would, however, in this case, be a substantial change in the distribution of income and there is a much greater likelihood, therefore, that the community's propensity to consume would be altered. If the marginal propensity to consume of owners were substantially lower than that of wage earners, then the net effect of the wage cut with rigid prices would be to reduce income and employment by weakening the community's propensity to consume.

Whatever is assumed about prices, however, Keynes' conclusion follows from his assumption, for the conclusion is that a change in wage rates cannot affect income and employment except through the propensity to consume or through one of the variables that determines the rate of investment. Reasons have already been given for believing that the assumption is not a satisfactory one. It can be defended, however, if the time lag between changes in business earnings and changes in their disbursements to owners is no greater than certain of the other time lags referred to in income analysis. And even if the Keynesian assumption is, in its extreme form, unrealistic, it suggests the way in which the alternative assumption that disbursements to owners are constant may be modified. If there is price flexibility of the first type, there is likely to be some induced decrease in owners' money incomes and if prices are inflexible, some induced increase as a result of changes in profits. The former would serve to limit the expansion of income and employment and to limit, also, the decrease in business profits or saving. In the latter case, changes in disbursements to owners would limit the contraction of income and the expansion of business saving. The limiting cases, with respect to this particular variable, are those in which business disbursements behave as they are assumed to in the *General Theory*. In that case, changes in wages will induce some redistribution of income and thus some change in the community's propensity to consume. This, in turn, may yield a change in employment as indicated above. But, otherwise, there can be no change in income and employment.

8. SUBSTITUTION EFFECTS

The foregoing discussion should have served to indicate how the general effects of wage changes can be analyzed under various assumptions about price behavior and about the time lags between changes in business profits and in business saving. As such it is an example of pure income analysis. This is, however, precisely its weakness for it is concerned solely with the income effects of initial changes in wage rates. Results can be achieved in a comparatively simple fashion because all relative changes in prices are disregarded so that substitution effects do not need to be taken into account. This limitation of income analysis is one of the reasons why Keynes' analysis and what has been called above the orthodox theory of wage changes lead to such different conclusions. The orthodox analysis invariably places greater stress upon substitution effects and its weakness is likely to be an inadequate treatment of income effects. The other, on the contrary, is apt to ignore substitution effects entirely. Clearly, then, if the controversy is much further resolved, some attempt must be made to bring them into the picture.

They are of two kinds. First: whenever wage changes give rise to relative changes in the prices of goods, there is likely to be a shift in the direction of expenditure, both by firms and by individuals from those products which have become relatively more expensive toward those which have become relatively cheaper. Such a shift represents the substitution of commodities in consumers' budgets. Second: whenever changes in wage rates together with consequent changes in prices alter the relative costs to employers of different types of input, that is, of different factors of production, producers will have an incentive to substitute the factor that has become relatively cheaper for that which has become relatively more expensive and thus to alter the method of production. This constitutes substitution of the factors of production employed in any productive process.

The total effects of price or wage changes cannot be arrived at simply by adding the pure substitution effects to income effects hitherto considered because, strictly speaking, the income effects of a price or wage change may include some shift in the direction of expenditure even if there has been no change in relative prices. Any change in the size of consumers' real incomes, for instance, is likely to involve some change in their composition as well. For this reason, it is incorrect to leave shifts in the direction of consumer expenditure or changes in the method of production out of account even when relative price changes are disregarded. In order to take account of all possibilities, it is necessary to consider both substitution that comes about as the result of changes in income and that which can properly be termed a substitu-

tion effect of relative price changes. The former type of substitution, however, is not particularly amenable to analysis and we shall have taken care of the more significant substitution effects if we consider those which would follow relative price changes.

The extent to which substitution of either commodities or factors of production will occur as the result of a wage change depends upon the behavior of prices. We have already had occasion to distinguish two types of price flexibility and, as a third case, complete price inflexibility. Before developing the implications of these alternative assumptions it will be well to give some further consideration to the relationship between price and wage-rate changes and, in particular, to the probability that a given change in wage rates will give rise to relative price changes, under various circumstances.

If price flexibility of the first type prevails, it is assumed that prices in any given firm move by the same percentage as marginal costs following a change in wage rates. Marginal costs consist of three elements: first, wage costs; second, supply costs (payments to suppliers); and third, what we may call materials costs or payments to other business firms. It goes without saying that the relative size of these various elements in marginal costs may differ widely from firm to firm and from industry to industry. It would appear, then, that, assuming price flexibility of the first type, a given change in wage rates in all firms would affect prices very differently depending upon the proportion of marginal costs made up of wage costs in the different enterprises. But this conclusion, although correct, is a little misleading as to the nature of relative changes in marginal costs.

Let us assume, for instance, that throughout the economic system wage costs and supply costs constitute the same percentage of marginal costs. Under these circumstances, regardless of the proportion of marginal costs made up of materials costs, a given change in wage rates will, under the assumption of the first type of price flexibility, lead to a percentage change in prices which will be the same for all products. One way of arriving at this conclusion is to assume that immediately following, say, a reduction in wage rates, all enterprises at once reduce their prices by the same proportion by which their marginal costs have been reduced. This initial price reduction will, of course, not be an equal percentage reduction for all prices, because in those firms where materials costs are a large element in marginal costs (retail stores, for instance) marginal costs will have been reduced by a much smaller percentage than in firms where wage costs and supply costs bulk large. Nevertheless, immediately after the initial price reduction a second series of price reductions must follow, for the initial price reduction will have constituted a reduction in materials costs

for virtually all enterprises. The second price cut will, from its very nature, be proportionately greater on the part of those firms in which materials costs are a large percentage of marginal costs. The second price reduction will, in turn, give rise to a third, and so on. It will be readily perceived that, by the time the series of price reductions has worked itself out, all prices will have been reduced by the same proportion as the sum of marginal wage and supply costs.

What this conclusion implies is that a general wage change can give rise to relative price changes under the assumption of the first type of price flexibility, only because wage costs and supply costs constitute different proportions of marginal costs. In general, price will be reduced by a larger percentage where the ratio of wage costs to supply costs in marginal costs is large, and conversely. Of course, if the ratio of wage costs to supply costs in marginal costs is high throughout the industries that sell raw materials and producers' goods and low in those industries that process them into consumers' goods, then marginal total costs are likely to move more than an average of wage and supply costs throughout the system. Under these circumstances, a high proportion of materials costs will be favorable to high percentage price changes, and vice versa. But it is still the relatively greater significance of wage costs than supply costs in certain areas that gives rise to relative price changes.

If price flexibility of the second type prevails, it will be remembered that prices are assumed to move by the same amount as marginal costs and, therefore, by a smaller percentage. This means that relative price changes can occur as the result of wage changes even if the proportion of wage costs and supply costs are the same throughout the economic system. For, under this assumption as to price behavior, a high proportion of wage costs relative to either materials costs or supply costs is favorable to proportionately larger price changes, rather than a high proportion of wage costs to supply costs alone. This may be readily perceived by following through the same type of reasoning employed above. It must not be lost sight of that it is the relative importance of these various elements in marginal rather than in average cost that is significant for the problem under consideration.

These results may now be applied to the main question at issue in this section: the nature and extent of two kinds of substitution effects distinguished above. The first of the two may be disposed of comparatively easily. Whenever there is any price flexibility in the economic system, it seems likely that, in those areas where marginal wage cost is a relatively large element in marginal total cost, prices will fluctuate relatively widely with wages and conversely. This implies that, following a wage reduction, those commodities for which the marginal wage

costs are a large proportion of marginal costs will become relatively, as well as absolutely, cheaper than before. There is certain, therefore, to be some substitution of those commodities for others. In the case of completely inflexible prices there will, of course, be no substitution of commodities whatever.

The question of the substitution of factors is a good deal more complicated, mainly because it is not, properly speaking, a short-run phenomenon at all. Nevertheless, it is not difficult to see what sorts of incentives for this type of substitution will result from wage changes. First: since it is assumed throughout that the prices of goods or services purchased by firms from suppliers remain constant, any wage reduction provides an incentive for the substitution of suppliers' products and services for labor, and vice versa. Second: no matter what type of price flexibility is assumed, prices will not move proportionately with wages. Even the first type of price flexibility involves only proportional movements of prices and marginal total costs. There will, therefore, always be some incentive for the substitution of labor for types of input which are themselves the products of business firms following a wage reduction.

One point of comparison between the two types of substitution may be mentioned. The substitution of goods in consumers' budgets is, on the whole, promoted by price flexibility. But the exact reverse is true of the substitution of factors whenever it takes the form of the substitution of labor for produced input or the reverse. The less flexible are the prices of capital goods and equipment, the more likely is it that wage changes will give rise to some substitution of factors. Thus, though both types of substitution operate in the same direction, one is more likely to be important if prices are relatively inflexible and the other if they are relatively flexible. No assumption can be made about prices, however, which eliminates the likelihood that one type of substitution or the other will occur.

The nature of substitution effects is sufficiently familiar so that it need not be further explored. As the above discussion should suggest, the principal difference between the substitution effects and the income effects of changes in wage rates is that the former are always in the same direction independently of what is assumed about prices or about the behavior of firms and individuals with regard to the disposal of their incomes, whereas the direction as well as the extent of the latter depend intimately upon such considerations. What is sometimes lost sight of, however, is that substitution effects themselves may give rise to further income effects so that their importance in particular cases may be very considerable indeed. Suppose, for instance, that there is a wage reduction with price flexibility of the second type.

Under this assumption, it will be remembered, the real incomes of wage earners will be reduced by the same amount by which those of suppliers and owners are increased so that, in the absence of a change in the proportion of income saved, there will be no changes in output and employment which can properly be called income effects of the wage cut. Nevertheless, there must necessarily be certain substitution effects. In the first place, prices of those products of which the marginal costs are largely wage costs will be reduced relatively to other prices. Therefore, there will be some shift in the direction of expenditure in favor of these goods. In the second place, an incentive will be supplied for the substitution of the services of wage earners for other types of input wherever it is possible. This suggests that, as a result of substitution effects, total disbursements to wage earners will be increased at the expense of disbursements to suppliers and at the expense, also, of disbursements by business firms to other firms. Insofar as this is the case, the increases in employment that follow directly from changes in the direction of expenditure and method of production are not the only substitution effects of the original wage reduction, for the increase in disbursements to wage earners may well give rise to a series of induced changes in consumption and output.

Strictly speaking, however, it is not certain that substitution effects of the first type, i.e., substitution of commodities, will involve an increase in disbursements to wage earners. Total wage disbursements will be increased by a shift in the direction of expenditure only if the ratio of marginal wage cost to price is higher in the firm toward which the shift occurs than in the firm or firms at the expense of which it is made. The fact that a high percentage of marginal total cost is made up of marginal wage cost is not sufficient by itself to insure that the shift in expenditure will increase total disbursements to wage earners.

Furthermore, just what are the induced income effects of an increase in disbursements to wage earners brought about by either type of substitution is by no means clear. If disbursements to wage earners are increased at the expense of those to suppliers, then there will be no induced effects unless the propensities to consume of the two groups differ. If it is brought about at the expense of disbursements by business firms, it is still more difficult to predict the results. In this case, although the increase in wage earners' incomes is certain to increase their consumption, the reduction in disbursements to other firms must immediately involve a reduction in the rate of production and in the rate at which income is disbursed by them. Just what will be the net effect on employment and on the incomes of the various groups involved depends, among other things, upon the rapidity with which disbursements by the firms in question to owners are affected. Never-

theless, it seems likely that the shift of income will be in favor of a group with a higher marginal propensity to consume. If it is assumed that firms' disbursements to owners are constant, or that they will vary only after a considerable time lag, the likelihood that this will be the case is increased. Insofar, then, as substitution in favor of disbursements to wage earners occurs at the expense of disbursements to business firms, it is likely to have favorable induced income effects.

Whatever the actual conclusion reached, however, enough has been said to suggest the way in which substitution effects can be analyzed and their repercussions followed through. What is more, since they are always in the same direction regardless of what is assumed about prices and since their nature depends only in a very secondary fashion on what is assumed about the rapidity with which disbursements are adjusted to receipts, the considerations here discussed would apply equally well to any of the cases developed above. In particular, it may be well to point out that what was called the Keynesian case might take on a rather different aspect if substitution effects were taken into account. Keynes assumes price flexibility of the first type and assumes, also, that disbursements to owners are altered instantaneously to correspond with changes in the net incomes of firms. Under these circumstances there will be no substitution effects if one other assumption is made. It is that wage costs are the only significant element in marginal costs, aside from disbursements to firms. Keynes does tacitly make this assumption because, in his analysis, there is no other factor of production or group of producers corresponding to what has here been designated as the category of suppliers. If wage costs and materials costs constitute the whole of marginal costs, and if price flexibility of the first type is assumed, it follows from reasoning employed above that a change in wage rates throughout the economic system cannot, by itself, give rise to substitution of either commodities or factors. Furthermore, in the Keynesian case, there is no redistribution of income as between wage earners and suppliers because the latter group is nonexistent and there is no redistribution of income as between wage earners and owners because disbursements to owners would be reduced as soon as business profits showed a tendency to decline following a wage cut. There is, therefore, no reason whatever, under Keynes' assumptions, to expect any shifts in the direction of expenditure.

Although he is, then, reasoning quite properly from the assumptions he makes, it should be clear that these are quite restrictive and that they define a very special case. So soon as another "original" factor of production (e.g., suppliers) is introduced into the picture, it is certain that relative price changes will occur and that some redistribution of income will take place as a result of a wage reduction. There is cer-

tain, therefore, to be some change in the direction of expenditure as a substitution effect of the wage change and there may be such shifts as the result of the change in the distribution of income. Furthermore, under these circumstances the wage change will furnish an incentive for substitution effects of the second type. Thus, even though the major Keynes assumptions are retained, it is hardly realistic to leave substitution effects out of account.

These considerations are of some importance because they help to explain the difference in conclusions reached by the Keynesian so-called "orthodox" theories of wages. Just as Keynes concentrated exclusively upon income effects, so economists of the other persuasion have been inclined, if not to disregard them, at least to underestimate their importance. Actually, of course, both sorts of effects have to be taken into account. This offers no difficulty in those situations where they are likely to be in the same direction. In other cases it raises a question as to which effects are likely to be the larger. This is not a matter which can be settled out of hand but one or two reasons may be adduced for believing that income effects are likely to be dominant. First: substitution of the second type (substitution of factors) is not, as pointed out above, a short-run phenomenon at all. In the long run, it is likely that prices will change in such a way as to reflect changes in wage rates. To the extent to which this occurs substitution of factors is not likely to be significant. Second: the very nature of the first type of substitution suggests that its net effect is not likely to be substantial.

A shift in expenditure in the direction of an industry in which marginal wage costs constitute a high proportion of marginal total costs is really (as pointed out above) a shift in the direction of expenditures in favor of wage earners at the expense of suppliers and other business firms. Undeniably, the very shift itself involves an increase in the employment of labor and a decline in the employment of the services of suppliers and of input purchased from other business firms but, for reasons given above, such a shift is not likely to have large induced income effects, that is to say, the increase in wage earners' consumption to which it gives rise is bound to be counterbalanced by decreased consumption on the part of suppliers, or decreased expenditures of all types on the part of business firms, or both. Thus, it is only the immediate or first effects of this type of substitution that are certain to be favorable to employment in the case of a wage reduction. The size of these first effects will depend upon price elasticities of demand and upon the extent to which wage costs constitute different proportions of total costs in different firms or industries. It seems scarcely likely, however, that a wage reduction of a given percentage could give rise to relative price changes sufficient to induce a change in disburse-

ments to wage earners of the same order of magnitude as the original wage cut itself. But unless it does so, the income effects are practically certain to outweigh the substitution effects.

9. SPECIFIC WAGE CHANGES

The place where substitution effects come into their own is in the case of a change in wage rates in a specific firm or industry or sector of the economic system. Unless complete price inflexibility is assumed, such a wage change must involve a relative price change as one of its most direct and immediate consequences. Therefore, there is certain to be a shift in the direction of expenditure in favor of the industry in which wage rates have been reduced. Furthermore, since only the prices of the products of the particular firm or industry in question will be altered as a result of the wage change, there is certain to be an incentive for the substitution of factors. This will, of course, be favorable to employment in the case of a wage reduction and unfavorable in the case of a wage increase, but it is by no means certain that the shift in expenditure resulting from the relative change in prices will operate in the same manner. When substitution of the first type (of commodities) occurs as the result of a general change in wages it is certain to involve a shift in expenditure toward industries in which wage costs constitute a larger than average proportion of the marginal total costs. Such industries are apt to be those in which the ratio of marginal wage cost to price is high. It is for this reason that the type of substitution is very likely to be favorable to employment following a general wage reduction. In the case of a specific wage cut, however, there is no reason why the ratio of marginal wage cost to price after the cut should be higher in the industry where wages have been reduced than in those which suffer from the shift in expenditure. What is reasonably certain is that both types of substitution will be favorable to employment (if they occur after a wage reduction) in the particular industry under consideration.

Although these substitution effects are likely to be of dominant importance in the case of a specific wage change, it does not follow that there are no income effects or that they can be disregarded. The reduction in wages itself constitutes, of course, an immediate cut in disbursements to wage earners. On the other hand, if the industry where the wage change is made is a consumers'-goods industry, and if there is any price flexibility whatever, the changes in prices that result from changes in wage rates will affect the price level of consumers' goods and thus the real incomes of all consumers. The income effects of a specific wage reduction with flexible prices may, therefore, be followed out by treating it as a small change in the level of wages in all consumers'-goods

industries accompanied by a small change in their prices. If there is price flexibility of the first type, and if disbursements to owners are held constant, a wage reduction will involve an increase in the real incomes of persons not earning wages and a smaller reduction in those of wage earners. The income effects, therefore, under these assumptions, would be favorable to employment. If prices are less flexible or if disbursements to owners change more rapidly, the income effects of a specific wage reduction may be neutral or unfavorable.

Perhaps the most significant comment that can be made about a specific wage reduction concerns the areas in which its effects are likely to be felt. The actual cut in wages itself constitutes, of course, a reduction in disbursements for the particular firm or industry in which it occurs but the income effects of the wage reduction (as distinguished from the associated price reduction) are certain to be rather widely diffused throughout the economic system. Turning to the price reduction, both its income and its substitution effects are certain to be favorable to an increase in output in the particular industry affected. Only its income effect, however, will be favorable in other areas. Putting these results together, it appears that in the particular industry both the reduction and disbursements and the income and substitution effects of the price reduction will be favorable to profits and employment. The effect of the reduction in wage earners' consumption upon the particular industry can be largely disregarded. Outside the particular industry, however, not only the effects of the reduction in disbursements to wage earners but also the substitution effects of the price change will be unfavorable. These two are certain to outweigh the favorable income effect of the price reduction.

These conclusions are by no means novel but they may serve to suggest why wage reductions are often expected to accomplish more than they will and why any analysis which emphasizes substitution effects alone supports such expectations. Since the unfavorable effects of a specific wage reduction are likely to be felt almost exclusively outside of the area where it is carried through and to be very widely diffused, it is not unnatural that attention should have been directed largely to its favorable effects upon income and employment. The above reasoning does not suggest that the unfavorable effects will be greater than the favorable effects but merely that there is a good reason why they are likely to be underestimated.

Little point would be served by the attempt to analyze the effects of a specific wage reduction in a capital-goods industry. The effects of the cut in disbursements to wage earners could be followed through in the same fashion, but if it is arbitrarily assumed that the rate of investment in real terms is constant, then the major favorable effects of wage reduction in this area are automatically assumed away.

10. CONCLUSION

The discussion contained in this paper does not begin to be a complete analysis of the effects of changes in wage rates. Nevertheless, it has, perhaps, been carried far enough to show the way in which income analysis can be used and to permit a few general conclusions about the effects of wage changes to be drawn.

Before embarking on this task, however, it may be well to point out the major limitations of the analysis as here presented. One is that the rate of investment in real terms has been taken as a constant throughout. The main reason for adopting this procedure has been to limit the scope of the analysis and make it manageable. The moment the assumption is relaxed, it becomes necessary to inquire into the whole matter of investment incentives and, then, to investigate the way in which a change in wage rates may affect the real rate of investment, either directly or via output, under all the various alternative assumptions that have here been employed.

Such an inquiry would turn up at least one question about investment incentives which could not be intelligently discussed within the limits of a single article. This concerns the relative importance of business profits and of the volume of output as factors in investment decisions. One of the conclusions that follows directly from the assumption that the planned saving of individuals increases with their incomes is that any change in wages and prices which is to affect income and employment favorably must reduce business profits and, on our assumptions, business saving. This means that there are almost bound to be certain types of wage and price changes which will strengthen one of the incentives to investment by leading to a short-run expansion of output but will, at the same time, weaken another incentive by reducing actual business earnings. Whether such a change will, on balance, be favorable or unfavorable to investment, it is impossible to decide on purely a priori grounds.

We are in the habit of reasoning that the marginal profitability of investment together with its cost is the determining consideration in any investment decision. It is quite possible, therefore, that, say, a reduction in wages with price flexibility of the first type might increase the marginal profitability of investment even though reducing the current volume of earnings. Should this be the case, such a wage reduction might have secondary repercussions upon employment via the rate of investment. On the other hand, it may be argued just as persuasively that such a wage and price reduction would reduce the liquidity of business firms, give rise to considerable uncertainty as to their ability to pay interest and dividends in the future, and thereby inhibit investment. Clearly, this is not a question that can be decided offhand. Yet it is one which has to be faced and resolved in some

fashion or other before it is possible usefully to relax the assumption that the real rate of investment is not affected, at least in the short run, by changes of the sort under discussion.

Even though this is true it must be pointed out that the assumption is always an unrealistic one. Not only is a change in output or in the level of business earnings likely to have an effect upon deliberate investment decisions one way or another, but it is common knowledge that no change in output can be brought about without some investment or disinvestment in inventories, a part of which may not be the result of deliberate decisions at all. In the course of an expansion of income, there is sure to be some disinvestment by merchants at various stages in the process because an increase in their sales will be followed only after the time lag by an increase in the rate at which they purchase merchandise. Conversely, any expansion process, once it has gained momentum, is almost sure to involve some deliberate stocking up which is likely to occur quite independently of any repercussions which the expansion may have upon deliberate long-term investment. If this is true of any expansion or contraction of income, it is all the more true of one which is set off by a change in wage rates and in prices. The change in wage rates constitutes a change in costs, and the expectation of changes in either costs or prices is almost certain to have an influence upon inventory policy. The assumption of a given rate of real investment is, then, highly unrealistic. Nevertheless, if the nature of the changes in investment that occur more or less automatically as a result of wage changes is understood, the assumption is not too damaging and it does serve to limit the scope of the discussion to a clearly defined part of the whole problem of wage theory.

What may be regarded as another major limitation of the analysis is that it contains no reference to the rate of interest or monetary policy. The reason is obvious. The assumption that the rate of investment in real terms is given makes it unnecessary to inquire into the effects of different monetary policies, for monetary policy affects employment by making borrowing of various sorts harder or easier and more or less costly, and the terms of borrowing do not enter into decisions concerning the rate of output except insofar as they enter into investment decisions. This conclusion may at first appear to be overstated for, as pointed out above, a simple change in output often necessitates further short-term borrowing or permits the repayment of loans, and anything which makes borrowing more or less costly will thus influence output decisions. This argument is valid enough but it is valid only because changes in output often involve investment or disinvestment. If they did not, they would not necessitate any change in the indebtedness of business firms. Since the blanket assumption

that investment in real terms remains constant enables us to disregard changes in stocks and inventories as well as changes in the volume of long-term investment, it also allows us to ignore the effect of interest rates and of credit policies upon employment.

In spite of these limitations, however, it may be suggested that the analysis contained in this paper calls attention to certain factors which have been largely disregarded in a number of current discussions of wage changes and, in so doing, suggests the direction in which income analysis must be elaborated and developed if it is to be a useful tool. The first factor of this sort is price policy. No argument is required to prove that the effects of wage-rate changes upon employment will depend in substantial measure on their effect on prices. Yet, as pointed out in various places, Keynes avoids the problem entirely in the *General Theory* by, first, assuming price flexibility, and, second, making an assumption about the nature of changes in business disbursements to owners which deprives price policy of most of its importance. When this latter assumption is employed, it will be recollected that a change in wage rates even with prices inflexible will affect output and employment only by changing the distribution of income and thus the propensity to consume. It has been suggested above that these assumptions are of a highly restrictive sort and that in the treatment of any general case different types of price policy must be considered.

Nevertheless, the prescription is easier to write than to follow. It is not difficult to set up (as has been done heretofore) one or two types of price flexibility and compare the results when these sorts of policies are assumed with those that would be achieved if prices were completely inflexible. But, in the real world, there is no unanimity in price policy throughout the economic system and such price policies as there are would not fall into any such simple categories. And there is still another difficulty. It may well turn out that, realistically speaking, there is extraordinarily little stability in the relationship between wages and prices, at least in the short run. In other words, it may be that at one time prices will respond very sharply to changes in wage rates and at another time very sluggishly if at all. This need not be due to deliberate changes in policy or even to a lack of consistency in the price policies of business firms. It is a result of the fact that prices depend upon many factors other than wage rates. Thus, a firm may have been on the point of an increase in prices because of the operation of factors solely on the demand side. A wage increase at that moment is far more likely to "cause" a price increase than under other circumstances. Furthermore, in such a situation a given wage change might be associated with a much larger percentage change in prices.

Yet it would not be quite accurate to say that the wage change itself had "caused" the whole price change. The central difficulty here, clearly, is that observed associations of price and wage changes cannot be taken as evidence that in every case the wage change "caused" the whole price change and that a wage change of the same magnitude will always gives rise to a price change of the same magnitude in the particular enterprise or industry where it has been observed. This does not, perhaps, damage the theory here developed because it may be possible to discover what price changes would be associated with given wage changes under normal or equilibrium conditions in particular industries. Nevertheless, the utility of the analysis for an explanation of observed phenomena may be seriously diminished by these considerations.

A second factor which, like price policy, has received too little attention in recent income analysis, especially of the Keynesian variety, is the existence of groups in the community which cannot logically be regarded as either business firms, wage earners, or receivers of fixed incomes. In this article the inclusion of the group of suppliers has been intended in part, at least, to correct this deficiency. It is not a minor one. Although wages and salaries constitute the largest element in income, the second is that designated entrepreneurial income. It represents mainly agricultural income and includes the incomes of small entrepreneurs in other lines of business. As suggested above, the receivers of such incomes are in a position more nearly analogous to that of wage earners than that of larger business firms functionally as well as socially. Their incomes represent payments received for the sale of products or services directly to business firms and consumers. They are not, as a rule, substantial employers of labor. Consequently, they are not likely to be affected directly by wage changes but they are likely to find the demand for their products and services enlarged whenever output increases. Furthermore, the manner in which they dispose of their incomes and divide them between saving and expenditure is likely to be closer to that in which wage earners dispose of their incomes than to that in which business firms do so. For all of these reasons it would appear that income analysis must be elaborated in such a way that it can take account of a larger number of significant economic groups than have frequently been distinguished from one another in the past.

A third and most important factor which has played all too little part in income analysis is the nature of the time lags between changes in receipts and resulting changes in expenditure by various individuals and institutions in the economic system. The prime example of a serious oversimplification in this respect is the Keynesian assumption

(often referred to above) that business firms may be treated as if they were owned by individual entrepreneurs whose consumption would respond to changes in their net incomes in much the same fashion as the consumption of wage earners or of any other individuals. Enough has been said to suggest that business saving is a very different kind of quantity from individual saving. Its residual character means not only that it is likely to fluctuate much more widely with income than the saving of individuals but also that the time lag between changes in the rate at which profits are earned and the rate at which they are paid out or withdrawn is likely to be far longer than time lags between changes in individuals' incomes and their expenditure. It is for this reason that business disbursements to owners were provisionally assumed in this article to be independent of the rate at which profits are actually being earned. As pointed out above, this is merely a way of assuming that the time lag between changes in earning and changes in withdrawals is sufficiently great so that a wage change will have had its impact on other quantities before withdrawals or disbursements of profits have been affected.

Not only is this worth mentioning as an example of a time lag, it is, in its own light, a factor of great significance in connection with wage changes or with price changes. Any wage or price change involves, at the moment when it occurs, a redistribution of income as between wage earners or consumers or both, on the one hand, and firms on the other. Its income effect must obviously depend, therefore, upon the rapidity with which the individuals who have been affected increase or reduce their expenditure and the firms affected alter their cost disbursements and their disbursements of profits. If the reaction patterns of the firms involved differ widely from those of the individuals, then the income shift can give rise, in the short run, to a relatively large increase or decrease in the total volume of expenditure. This constitutes the income effect of the wage or price change. A substantial part of it may be temporary if, for instance, the firms or individuals benefited by the shift ultimately disburse as large a flow of funds as those at whose expense the shift has been conducted. Nevertheless, these temporary income effects are quite large enough so that they need to be taken into account.

In the terms of the quantity theory these effects could be described as changes in velocity or in income velocity. The only difficulty in stating the matter in this fashion is that it does not cast much light upon the motives and decisions which underlie the phenomenon. In whatever form it is stated, however, this analysis of the relationship between time lags and income effects indicates the importance for wage and price theory of corporate dividend policy and of the policies

of owners of small business firms toward the withdrawal of profits.

If Mr. Keynes' disregard of the time lags between changes in business receipts and changes in profit disbursements is a particularly serious example of the failure to give due attention to time lags, it is by no means the only one, nor is any income analysis made perfect merely by remedying this fault. Ideally, it should be possible to take account of differences in time lags between receipts and disbursements for all of the major groups in the community. However it may be accomplished, income analysis, if it is to be useful for dynamic analysis, must be so handled that the significance of such time lags is brought out. Mr. Keynes is not interested in them because his primary concern is still with equilibrium theory of a sort and with the way in which changes in prices, or in monetary policy, or in investment opportunity will affect employment after they have had time to work out their effects fully. This is, of course, a legitimate and useful type of inquiry but it has serious limitations and may be highly misleading if applied to certain sorts of problems. The reason is that if certain of the effects of an initial change appear promptly and others only after a long time lag, then the initial effects may have time to set in motion a course of events which will modify the delayed effects or prevent them from appearing at all. It may well be that an initial change will give rise to certain immediate movements in one direction which would be reversed when and if more delayed effects had time to appear. Whenever this is true, it is highly misleading to analyze the results of the initial impulse by discovering what its effects would be when fully worked out, disregarding the path that would be followed as they developed and the induced effects that might flow from the temporary situation. If, for instance, a wage cut with price flexibility of the first type will have the immediate effect of expanding income and employment and if this immediate effect can be expected to last only so long as business firms do not reduce dividend and interest payments following the decline in their earnings, it might be reasoned that the ultimate effect of the wage cut will be nil. On the other hand, it is quite conceivable that the temporary change in income and employment would last long enough to induce, say, a large increase in consumption financed through installment purchases or a substantial increase in investment. Here, the effect of the induced changes in the propensity to consume or the rate of investment would be to reinforce the initial effects of the wage cut and, probably, to overcome its delayed effects entirely.

Such possibilities as these cannot be disregarded if income analysis is to be used for anything other than the analysis of equilibrium situations. The real significance, then, of the assumption that business

disbursements to owners are constant is that it enables us to examine certain effects of a wage change that may be expected to appear very promptly and to distinguish them from other effects which will be delayed. There are, no doubt, many other types of time lags which need to be taken into account in much the same way. Income analysis will, it is to be hoped, develop in such a way that they can be taken into account without at the same time turning into the sort of period analysis that depends upon very explicit assumptions about different types of time lags and is, in consequence, wholly lacking in generality.

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EMPLOYMENT, INVESTMENT, AND THE MULTIPLIER*

By MONTGOMERY D. ANDERSON

I

LET P_0 be the money value of the rate of sale of some given good by all producers in a closed economy at a given time while W_0 is the current value of personal services and P_1 is the current value of other goods being purchased to produce the stream of finished goods selling for P_0. Then

$$P_0 = W_0 + P_1 + d_0. \tag{1}$$

The term d_0 represents the excess or deficit of rate of sale of finished product over the current value of the goods and services being bought to maintain the stream of products selling for P_0. W_0 and P_1 are not rates of expense but current rates of purchase. The subscript, 1, is placed on the second P in formula (1) to indicate that this amount of money is being paid for a stream of goods at a stage of the industrial organization one step removed from that at which the price P_0 is being paid.

The stream of goods priced by P_1 are themselves the product of labor and still other goods purchased at a stage of the productive process removed once again from that of the "original" stage where the goods priced by P_0 were sold. Hence the value of P_1 may be stated by the following:

$$P_1 = W_1 + P_2 + d_1, \tag{2}$$

where the significance of the terms should be obvious from the analogous terms in (1). Likewise, one may write,

$$P_2 = W_2 + P_3 + d_2, \tag{3}$$

and so on. Thus the value of goods employed at stage zero may be "cross-cracked," so to speak, into labor values and goods values through an indefinite number of stages of production, or until practically their entire value is stated in terms of wages only. The process may be repeated until the value unaccounted for by wages, say P_{r+1}, becomes practically zero. By substituting formula (3) in (2) and (2) in (1), etc., the following result is obtained:

* A few of the equations appearing in the first section of this paper have been published previously in *The Southern Economic Journal*. They are reproduced herein (with permission of the editors of that journal) because they are so vitally necessary to an understanding of the main argument of the present paper.

$$
(4)\begin{cases}
P_0 = W_0 + W_1 + W_2 + \cdots + W_r + d_0 + d_1 + d_2 + \cdots + d_r, \\
P_1 = \quad W_1 + W_2 + \cdots + W_r \quad + d_1 + d_2 + \cdots + d_r, \\
P_2 = \quad W_2 + \cdots + W_r \quad + d_2 + \cdots + d_r, \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
P_r = \quad + W_r \quad + d_r.
\end{cases}
$$

Thus the wage payment, W_r, appears many times in the guise of a commodity, so to speak, while the wage payment, W_1, appears twice in the exchange process in the guise of a commodity, being a part of the value of commodity 1 and indirectly a part of the value of commodity 0.

Let $X=1, 2, 3, \cdots, r$ represent the subscripts of P, W, and d. Then in the case where $P_{r+1}=0$, as in formula (4) it can be shown that

$$
(5) \qquad \sum_{x=0}^{r} P_x = \sum_{x=0}^{r} (X+1)W_x + \sum_{x=0}^{r} (X+1)d_x
$$

$$
(6) \qquad = \sum_{x=0}^{r} XW_x + \sum_{x=0}^{r} W_x + \sum_{x=0}^{r} Xd_x + \sum_{x=0}^{r} d_x.
$$

Other series of prices may be generated starting with other products as the zero stage of production, whose prices might be labeled P_0', P_0'', etc. For each of these series a formula like (5) will be obtained, which expresses each as a weighted sum of wage rates and of differences, d_x. The sum of all the series combined will equal the total rate of sale of goods and services of all traders provided duplications are avoided in generating the series. That this last may be true there is no necessity for defining what is meant by a "zero" stage of production, and no need to develop romantic distinctions between "capital goods" and "finished goods." The "zero" stage is just wherever one chooses to make a beginning. The only requirement is that, whenever a series should happen to "collide" with one already developed and included in the grand total, the anterior and posterior portions should all be treated as a single series, and the posterior part should not be counted again.

This process of generating series and summing them may be continued until every price being paid for goods and personal services at a given time has been counted, and counted only once on the left-hand side of the equations. At this point it must follow that $\Sigma d_x=0$. While the current rate of sale of goods and services by any individual producer may be greater or less than his current rate of expenditure, it is necessarily true that the total rate of sale of all traders at a given time must be exactly equal to their total rate of expenditure.

Since there is presumably no correlation between X and d_x, and since the mean value of d_x is zero, it can be shown that $\Sigma X d_x = 0$. Hence equation (6) reduces to

$$\sum P_x = \sum XW_x + \sum W_x, \tag{7}$$

when the summation sign is understood to stand for a double summation, once with respect to X, for each generative series like formula (5), and once again with respect to each of the sums thus obtained from all the initial points of departure. The two sums on the right both converge, hence the sum of the prices P_x will be a definite, manageable quantity even when r is a very large number.[1]

Formula (7) expresses the total rate of sale of all goods and services in a closed economy as a function of the total wages and the "stages" of production. The formula may be revised to read[2]

$$\sum P_x = \left[\frac{\sum XW_x}{\sum W_x}\right]\sum W_x + \sum W_x. \tag{8}$$

To find the value of all goods traded exclusive of labor, ΣW_x must be subtracted from ΣP_x, which yields the result:

$$\sum P_x - \sum W_x = \left[\frac{\sum XW_x}{\sum W_x}\right]\sum W_x. \tag{9}$$

Thus the total rate of sale of goods only is equal to a certain coefficient times the total rate of wages. Let this coefficient be designated by the letter μ, and let $\Sigma W = \phi$, while the rate of sale of goods only is represented by S. Then (9) becomes

$$S = \phi\mu. \tag{10}$$

The coefficient μ in formula (10) gives the (weighted) average number of entrepreneurial intermediaries between the buyers of goods and the buyers of the labor out of which goods are made.

Let $Z = S + \phi$, so that Z is the rate of sale of all goods and personal services (wages) in a closed economy. Then by substitution in formula (10) it follows quite simply that

$$\phi = \frac{Z}{\mu + 1}. \tag{11}$$

[1] The convergence is indicated by the fact that while W_x may increase spasmodically, the general drift must be in the direction of zero, as the original P_0 is split repeatedly into two parts.

[2] In formulas (5), (6), (7), and (8) net rents are omitted entirely, i.e., they do not appear on either side of the equations; for if the payment of rent is not a purchase of goods or labor the receipt of it is not a sale of goods or labor. The matter of rent does not impair the validity of the formulas in the least.

Moreover, the rate of sale of all goods and services for periods of a year or more in duration is approximately the same as the total rate of payments for all goods and services. For such periods Z may be regarded as either total sales or total payments.

Let the term V be defined as the ratio of Z to M, the total amount of money in circulation, including both time and demand deposits, so that $Z=MV$. So defined V becomes a measure of the average rate of turnover of all money[3] in exchange for goods and services. Substitution in equation (11) yields the result that

$$\phi = \frac{MV}{\mu + 1} . \tag{12}$$

Let N represent the equivalent number of persons employed in their respective occupations at "full," or standard time, for the duration of the interval over which the rate of wages is measured. According to this concept two persons employed half time for, say, a year, would be counted as equivalent to one person employed for a year, and so on. Let W represent the average wage earned by a person working full time for the duration of the interval over which the total rate of wages is measured. Then, as a matter of definition, $W=\phi/N$. By substitution in formula (12) it follows that

$$N = \frac{MV}{(\mu + 1)W} . \tag{13}$$

Let X be the average product per worker per interval of time, measured not in money but in goods and services,[4] and let P be an index of the general price level, so that by definition $X=W/P$.[5] If the average

[3] There should be no need to pause long for discussion of the utterly stupid objection that "time deposits do not circulate in exchange for goods." They do so circulate, indirectly, and the statistical concept V as above defined has a very definite economic significance. In fact, the average rate of turnover of all deposits is for most purposes a more significant statistic than the rate for demand deposits alone.

[4] The term X will hereinafter be referred to as the "average productivity of labor."

[5] The concepts X and P are difficult to define precisely but they have a very real meaning for the layman as well as the economist. The intricate problem of their precise evaluation need not detain the present investigation. For a comprehensive summary of the modern theory of index numbers see Ragnar Frisch, "The Problem of Index Numbers," ECONOMETRICA, Vol. 4, January, 1936. p. 1. Also compare A. A. Konüs, "The Problem of the True Index of the Cost of Living," *ibid.*, Vol. 7, January, 1939, p. 10. Since W is measured in dollars and P is an index number, the quotient X may be thought of as so many *stable* dollars, i.e., dollars of constant purchasing power, or it may be regarded as an index of product per worker measured in goods.

worker produces W dollars' worth of goods and services per year which sell for an average unit price of P, then the average worker's "real" product is obtained by dividing P into W. This statement treats the worker's money wage and the money value of his product as identical, which in point of fact they are for workers as a whole, if the wage rate is taken to be not the labor cost of the goods currently being sold, but the rate of wages currently being paid.[6]

Substituting for W in formula (13) gives the following comprehensive formula for total employment in a closed economy:

$$N = \frac{MV}{(\mu + 1)PX}. \tag{14}$$

This formula states the equivalent number of persons given full employment at any given time as a function of five other variables, viz., the amount of money in circulation, its rate of turnover, the coefficient μ, the price level, and the average productivity of labor.

A convenient differential form of this equation is given by

$$\frac{N'}{N} = \frac{M'}{M} + \frac{V'}{V} - \frac{\mu'}{\mu + 1} - \frac{P'}{P} - \frac{X'}{X}, \tag{15}$$

where the primes represent rates of change with respect to time.

II

It is important to note from formula (15) that, contrary to the compensation theory, an increase in the productive efficiency of labor does not tend to increase employment but tends to decrease it. This finding with respect to the production of goods in general has received verification by Tinbergen and de Wolff with respect to the normal production of "consumption goods" only.[7] Formula (14) may be rearranged to show why it is true that in the production of exchange values in general an increase in labor productivity is not compensated by the use of more labor and less capital.

The numerator of formula (14) is by definition equal to the total rate of sale of goods and services. If K is a properly computed period of turnover of all goods (including the constructive turnover of fixed capital as it is charged to depreciation)then KMV is the money value of the total wealth of society, and KMV/P is the "real" value of total

[6] This principle has been explained by the writer in a previous paper. Cf. "Dynamic Theory of Wages," *The Southern Economic Journal*, Vol. 6, July, 1939, pp. 43–55.

[7] Cf. "A Simplified Model of the Causation of Technological Unemployment," ECONOMETRICA, Vol. 7, July, 1939, p. 193.

wealth. Let ψ represent the last-named quantity. Substituting ψ in formula (14) gives

$$N = \frac{\psi}{K(\mu + 1)X}. \tag{16}$$

It should be rather obvious from an inspection of this formula that if labor productivity, as represented by X, should increase there cannot be a simultaneous increase in the employment of labor and a compensatory decrease in the amount of wealth unless there should also be a percentage decrease in the value of the product, $K(\mu+1)$, more than equal to the numerical sum of the percentage changes in labor productivity and amount of wealth in existence. It is highly improbable that any decline in the normal value of the product, $K(\mu+1)$, of such proportion could occur. As shown by the writer in the reference cited above, the normal value of the coefficient μ is virtually constant. In the long run, moreover, it would seem that the term K should be positively correlated with time or not correlated with it, which in either case would prevent any normal decline in the product, $K(\mu+1)$.

Now grant the writer's contention that, in general, consumption goods produce an income indirectly and should therefore be considered a part of the total capital of society.[8] In that case wealth and capital are interchangeable terms in formula (14), and it becomes clear why more labor and less capital is not utilized when labor becomes more productive. This would not be possible without a very considerable decline in the average period of investment, or average period of turnover of all capital.

Now, the term X is defined as the "real" product of a laborer working "full" time during an interval of solar time. So long as the duration of "full" time is held constant an increase in labor productivity as measured by X will represent an increase in the actual efficiency of a laborer *while working*. If the duration of full time is not constant, however, then an increase in productivity as measured by X might be due solely to an increase in the standard number of hours worked per week or per year by the average workman. This would not be an increase in labor productivity as it is ordinarily conceived.

[8] Take bread consumed by a workman for example. He sells the product of the bread in the form of muscular energy and the bread therefore yields him an income indirectly. Most of the goods commonly classed as capital yield an income only indirectly as, for example, fuel thrown in the steam boiler of a factory. The fact that a worker may eat bread and then go on a strike so that the bread is not productive does not alter the typical aspect of the matter. Is oil-well casing any less capital in general because all too frequently some of it is abandoned in a dry hole?

Such ambiguity in the significance of X may have its disadvantages, but it has also great advantage in that it calls attention to the fact that the explicit effect upon employment of an increase in labor efficiency is the same as an increase in the length of the work week, or work day. This means, in theory at least, that increases in labor productivity caused by improved technology might be largely compensated by corresponding reductions in standard time so that no unemployment would result.

This conclusion has also received independent verification by Tinbergen and de Wolff. They find that a reduction of the work week from 48 to 40 hours has, in the United States, a *normal tendency* to increase employment eight to ten per cent if weekly wages are held constant and about twenty-six per cent if hourly wages are kept constant. These same results follow from the theory of employment presented in this peper, but the results are obtained in this case with much greater simplicity, because the authors just quoted employ twenty-eight variables in their analysis, whereas formula (15) above employs only six. The application of formula (15) to compute the effect of a reduction in hours runs as follows:

There was no appreciable trend in the velocity of all money in the United States in the half century prior to the Great Depression. Therefore V'/V may be set to equal to zero. Since the normal value of μ is also constant, $\mu'/(\mu+1)$ may also be set equal to zero. If weekly wages are to be held constant then the product PX must be invariable, provided the variable X is stated in terms of output per worker per week, for in such case PX will state the value of weekly wages per worker. Hence the stated condition requires that the sum of P'/P plus X'/X shall equal zero. Therefore the value of all the terms on the right-hand side of formula (15) reduces simply to that of the first term: M'/M. In other words the normal percentage gain of employment with constant weekly wages should equal the normal percentage rate of gain of the amount of money in circulation, regardless of any change in the standard number of work hours per week. Reliable data for total money in circulation in the United States extending back to 1850 are not available so far as the writer knows, but a reliable series may be obtained back to 1880. According to this series the total amount of money (including bank deposits) in circulation in the United States had a normal growth of about 6.8 per cent per annum prior to the Great Depression.

Hence the estimated normal value of N'/N should be 6.8 per cent regardless of any change in the work week, provided wages per person per week are held constant. This compares rather well with the 8-per-cent figure obtained by Tinbergen and DeWolff, when one considers

the rough approximations made both by them and by this writer.

If hourly wages are held constant, while the work week is reduced, the application of formula (15) is a little more complicated. If the work week is h hours long and X is stated in terms of weekly intervals, then average hourly wages are equal to PX/h. If this expression is kept constant while h is reduced from 48 to 40, or about 18 per cent, then the requirement is roughly that

$$\left[\frac{PX}{h}\right]' \frac{h}{PX} = 0 \quad \text{while} \quad \frac{h'}{h} = -0.18. \tag{17}$$

In other words,

$$\frac{P'}{P} + \frac{X'}{X} - (-0.18) = 0, \tag{18}$$

or

$$\frac{P'}{P} + \frac{X'}{X} = -0.18. \tag{19}$$

Substituting this expression in formula (15) and again holding V and μ constant yields the result that

$$\frac{N'}{N} = 0.068 - (-0.18) = +0.24. \tag{20}$$

Once again a good agreement with Tinbergen and de Wolff is obtained —but with a vast deal more of logical simplicity and statistical ease. Their estimated normal increase in employment under the stated condition is "26 to 28 per cent."[8a]

III

The theory of wages and employment set forth in this paper may be made to yield a formula for the multiplier if one will concede the writer's contention that even consumption goods as a general rule yield income indirectly and therefore should be counted as a part of the total capital of society. In that case the rate of investment for a closed economy plainly is given by the simple formula:

$$\gamma = Y'R, \tag{21}$$

where γ is the rate of investment, Y' is the rate of increase of wealth or capital, and R is the average rate of turnover of all wealth or capital.

[8a] It is assumed tacitly in the foregoing paragraph that the *normal* number of weeks worked per year remained constant during the period in question.

Plainly if new capital is coming into existence at the rate Y', and the new (and old) capital is turning over at the rate R, then new capital is being purchased, i.e., money is being spent for new capital at the rate $Y'R$.[9] (By definition $R=Z/Y$ where Z is the total rate of sales and Y is total capital.)

Moreover the rate of net income earned by all capital (including net rent of land, etc.) must be the same thing as the rate of increase of capital, by definition. (Contractual interest earned and paid may be neglected for society as a whole since they are offsets.) Hence Y' is the net income of capital or "unearned" part of the national income.

In the initial section of this paper it was shown that the total rate of wages paid in a closed economy is given by the formula, $MV/(\mu+1)$, where the symbols have the significance explained in that section. It has been shown further that Y, total capital, may be expressed as the product, MVK, where K is the reciprocal of R, i.e., where K is the average period of turnover of all wealth or capital. Hence the total rate of wages is given by

$$\phi = \frac{Y}{K(\mu+1)} = \frac{YR}{\mu+1}. \tag{22}$$

The multiplier may be defined as the number of dollars of national income associated with each dollar of investment. In that case it will be equal to the sum of wages and net income of capital divided by investment, or, in the language of symbols,

$$\beta = \frac{Y' + Y\,R/\mu+1)}{Y'R} = \frac{1}{R} + \frac{Y}{Y'(\mu+1)}, \tag{23}$$

[9] The rate of investment is here conceived to be the rate at which money is being put into "new" capital, i.e., the rate of payment for "new" capital. In so defining the term the writer is adopting the definition of J. M. Keynes in his *General Theory*, in so far as it is possible to determine what Mr. Keynes really does mean by the phrase, "to invest." The Keynesian conception is, to be sure, not entirely clear. As R. G. Hawtrey has truly said, "When Mr. Keynes speaks of the 'inducement to invest' we must discover the meaning of 'to invest'." (Cf. *Capital and Employment*, p. 177.)

The present writer's definition of investment is also the same as Hawtrey's definition of *active* investment. Mr. Hawtrey writes, "We have defined active investment to mean the voluntary acquisition of items of unconsumed wealth in the expectation that they will be remunerative." Voluntary acquisition means purchase, in an exchange economy, if wealth is valued at cost less depreciation, as is customary; because increments of value in wealth which is already owned are not recognized on the books until the moment of sale. Wealth acquired for the sake of remuneration must mean capital. If unconsumed capital is any different from capital in general it must be different in the respect that it has been newly created. Thus Hawtrey's definition reduces to the writer's more simple language by substitution of synonyms.

where β is the multiplier. Y/Y' is recognized as the reciprocal of the average percentage rate of return to capital, which may be designated by the letter r. Then formula (23) becomes

$$\beta = \frac{1}{R} + \frac{1}{r(\mu + 1)} \cdot \tag{24}$$

Statistical evaluation of this formula calls attention sharply to a theoretical point of major importance; namely, that the rate of investment does not have the same time dimension as either the rate of net income of capital or the rate of wages. The rate of investment is correctly stated as so many dollars per time interval per time interval, whereas the rate of net income and the rate of wages are both stated as so many dollars per time interval. The investment rate has a double time dimension whereas the other two concepts have only one.[10] Failure to realize this creates endless confusion and complication in economic thought and this confusion is only compounded by the introduction of such clumsy devices as Robertson's "indivisible particle of time." The invention of such devices probably signifies a dim percertion that all is not well without a realization of precisely where the trouble lies.

There can be no doubt whatever about the writer's contention—and it must be faced squarely by fellow econometricians in the long run—that the rate of investment has a double time dimension. Reference to the simple formula for rate of investment given above shows that the Y term is stated as so many dollars per time interval while the R term is so many times per time interval. The product of the two terms therefore has a double time dimension.[11]

[10] The double time dimension might be avoided by defining the rate of investment as the rate of purchase of new capital *for the first time*. By such definition the investment rate would be identical with the rate of increase of capital in an exchange economy, where such increases are not recognized until they are validated by a sale. Then the term, rate of investment, would seem to be more or less useless and redundant.

[11] The *amount* of net income and wages earned in a given interval of time does not have a time dimension but the *amount* of investment has a single time dimension in strict logic, although in practice this dimension may be concealed by the integration formula used by the computer. The *amount* of investment is properly conceived as the integral of the rate over some definite period of time just as the amount of wages paid would be a definite integral of the rate of wages. The formula for amount of investment $G(t)$ in the period of time ϵ ending at t is given by

$$G(t) = \int_{t-\epsilon}^{t} R(t) \frac{dY}{dt}\, dt. \tag{i}$$

This formula is seen to be the integral of the product of two rates per year. It will be expressed in dollars *per year*. So conceived the amount of investment rep-

Nor is this plurality of time dimension in any way a consequence of the writer's pecular inclusion of consumption goods as a part of capital, as a moment's reflection will show.

Hence the multiplier is not merely a numerical ratio as it would seem to be at first blush from a purely phonetic definition. It has a time dimension, obtained when a variable having a double time dimension is divided into a variable having only one time dimension. Hence also the numerical value obtained for the multiplier by statistical measurement is dependent in part upon what time interval is adopted by the appraiser as a modulus.

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resents the *amount* by which the rate of production of exchange values has been increased within a given period *by means of* the introduction, or creation, of new capital.

In practice the statistician usually gets rid of the bothersome time dimension of investment by adopting a peculiar conception thereof. By this conception the amount of investment in the period ϵ is taken to be not the integral of a product as in (*i*) but rather the double integral given by the formula,

(ii) $$G(t) = \int_{t-\epsilon}^{t} \int_{t-\epsilon}^{t} R(t) \frac{dY}{dt}\, dt.$$

In other words common practice is to compute the amount of *so-called* "new" capital which was purchased in the given finite interval. The amount of investment so reckoned will not have a time dimension for it will be just so many dollars, but the numerical evaluation of such a reckoning for a given period of time tends to be a function of the square of the interval of integration arbitrarily selected. To put it differently, the formula does not give consistent results as may be seen very readily if dY/dt and $R(t)$ are assumed to be constant.

In that case the amount of investment for a period of $\epsilon/10$ would be only one one-hundreth of the amount for the period ϵ, which is plainly inconsistent, for as a matter of common sense the former amount should be of the order of magnitude of one-tenth of the latter.

In other words the amount of investment by the ordinary reckoning depends upon the length of time over which newly created capital is assumed to keep "new." The result is meaningless unless this interval of integration is stated. It is not sufficient to say that the amount of investment was so many dollars in a given year. One should add the phrase: "provided new capital stays 'new' a year."

This curious imaginary interval may be termed "an indivisible particle of time," if you like. But giving it a fanciful name is only a logical subterfuge to conceal what should be frankly recognized as a time dimension.

Of course, where ϵ is a unit of time and dY/dt and R are both constant formulas, (i) and (ii) yield the same numerical results although their dimensions are different. Since the normal values of dY/dt and R do tend to be constant for yearly intervals, and the calculations ordinarily made by use of formula (ii) are for unit (yearly) intervals the evaluation of investment commonly given is numerically correct, and the lack of the missing time dimension goes unnoticed. If the amount of normal investment is really, say, five billion dollars per year it will be ordinarily reckoned as five billion dollars.

This is certainly true if the multiplier is defined as above, i.e., as the ratio of national income to investment. The time dimension may be eliminated if the multiplier is defined as the ratio of the *increase* of national income to investment. But in this case the value of the ratio becomes a small fraction having a normal value of about 20 per cent for the United States, which is far from the value commonly quoted by econometricians. In this case the time dimension disappears but the term "multiplier" becomes an egregious misnomer. On the contrary the concept should then be called "the reducer," or some such name, for it would give the fractional part of a dollar of investment which appears as a part of the increase of national income.

If the multiplier is defined as the ratio of increase of national income to increase of investment, the time dimension reappears in the multiplier.

The commonly cited numerical value of the multiplier is in the neighborhood of 2.00. This value will be obtained approximately only if a yearly time interval is used to calculate the multiplier; i.e., if it be defined as the numerical ratio of yearly national income to yearly investment. The normal rate of turnover of all wealth in the United States per year was about 1.00 before 1929, and the normal percentage rate of net return to capital was slightly less than 7 per cent per annum, while μ tends to be about 6. Hence the normal multiplier was given approximately by formula (24) as

$$\beta = 1.00 + \frac{1}{7(0.07)} = 3.00. \tag{25}$$

Thus every dollar invested per year per year tended to produce, or be associated with, one dollar of net profit per year and two dollars of wages per year. The double time dimension may sound awkward, but it belongs in the statement; otherwise it is not exact. Indeed, without any statement of time dimensions at all the multiplier is meaningless. This may be shown very quickly by showing that with a monthly interval of reference the multiplier would be 36, or twelve times as great as above, viz.,

$$\beta = \frac{1}{\frac{1}{12}} + \frac{1}{7\left(\frac{0.07}{12}\right)} = 36. \tag{26}$$

By *mixing* time intervals one may obtain almost any value for the multiplier. Suppose a *monthly* interval is used for measuring R, and that the rate of interest r is stated with reference to a yearly interval. In that case β would be evaluated as follows:

$$\beta = \frac{1}{1/12} + \frac{1}{7(0.07)} = 12 + 2 = 14. \tag{27}$$

This is a preposterous value of course. No one would consciously mix the intervals of reference but it is easy to do this unconsciously. It is interesting to note that Tinbergen and de Wolff obtain this preposterous value of 14 for the multiplier as a result of their analysis, and that this result might have been obtained by confusing time dimensions.

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A NOTE ON PARETO'S THEORY OF PRODUCTION

By H. Neisser

In an article, "Marginal Productivity and the General Pricing Process," (*Journal of Political Economy*, October, 1929) the late Professor Henry Schultz reopened the discussion about the comparative advantages of the Walrasian and the Paretian approach to the theory of production. Schultz himself unhesitatingly decided in favor of Pareto. Walrasians, on the other hand, usually have been content to refute what seemed to them unwarranted criticism, and to develop further the theory of the production function. It is not the intention of this paper to take sides in the dispute or to discuss all problems of the general theory of production. It only wants to call attention to certain unnecessary complications and theoretical difficulties in Pareto's approach.

The general theory of equilibrium practically consists of five sets of equations (disregarding Walras' interest-rate and monetary equations). Of these five sets, set (1), demand equations, set (5), supply equations for productive services, and set (2), stating the equilibrium condition, price = average costs, need not be discussed here. Our interest centers around set (3), establishing the "full utilization" of available productive services, and on set (4) indicating the entrepreneur's choice of the "technique of production"; with Pareto, it is intimately connected with set (3). It is for set (4) that Paretians claim a definite superiority over Walras.

I

Walras' set (3), the condition of full utilization, follows directly from his definition of a "coefficient of production": $a_{ij} = a_{ij}(o_i, x_{ij}) = x_{ij}/o_i$, and is only another form of defining X_j:

$$X_j = \sum_{i=1}^{n} a_{ij} o_i; \tag{3a}$$

o_i denotes the n quantities of goods produced, X_j the supply of r production services, and x_{ij} their input.

Pareto, on the other hand, prefers the more general form,

$$X_j = F_j(o_1, \cdots, o_n), \tag{3b}$$

where F_j denotes some function, the nature of which will be examined below. Instead of the set of nr Walrasian coefficients of production, Pareto introduces the nr partial derivatives of the system (3b) as "Paretian coefficients of production" ($\bar{a}_{ij}$).

Let us examine first the alleged disadvantages of the Walrasian

approach. The objections are based on the following statement: "To say, as we did, that, in producing a good, one can utilize more or less of some productive service, e.g., of land, provided one utilizes less or more of some other services, e.g., of capital goods or labor, implies that the coefficients of production a_{i1}, a_{i2}, a_{i3}, $\cdots$ are variable and interrelated by a *production equation*,

$$\phi_i(a_{i1}, a_{i2}, a_{i3}, \cdots) = 0, \tag{6a}$$

in such a way that if one or the other of the coefficients, a_{i1}, decreases, some of the other coefficients, a_{i2}, a_{i3}, $\cdots$ increase."[1]

From the Paretian point of view, it is inadmissible to treat all coefficients as "compensatory," i.e., as interrelated by an equation like (6a). Pareto introduces, besides these compensatory coefficients (relating to what we call now substitutional factors), the coefficients which "are constant or almost constant" and the coefficients "which vary with the quantity of output"[2] both referring to the "limitational" factors. The distinction is of fundamental importance, though of course Walras' set (3a) is not invalidated by it. Less important is a second objection, coming from Schultz, that, by introducing equation (6a), Walras unnecessarily restricted the form of the production function, which gives output as function of input, $o_i = \Phi_i(x_{i1}, x_{i2}, \cdots, x_{ir})$. Schultz indeed proved[3] that (6a) implies a first-degree homogeneous function, i.e., absence of advantages of large scale. But this fact[4] is due solely to the restricted form in which Walras stated the functional relationship, and which has nothing to do with the compensatory nature of the Walrasian coefficients, let alone with the definition. *Prima facie*, there is no reason to assume that there exists one and the same functional relationship between Walras coefficients for *all* values of $x_{i1}, \cdots, x_{ir}$. If one does not want to impose any restriction on the production function, one can simply state the condition of substitutability, for some or all of the coefficients, by

[1] *Éléments d'économie politique pure*, édition définitive, 1926, §325. In the translation, Walras' notation is replaced by the notation used in the present paper. For the translation of Walras' terms "rente" and "profit" see *Éléments*, §§170–172.

[2] Vilfredo Pareto, *Manuel d'économie politique*, 2nd. ed., Paris, 1927, Appendix, Sec. 101.

[3] *Loc. cit.*, Appendix II.

[4] The converse theorem also holds true: Homogeneous functions of the first degree lead to a Walrasian relationship between the coefficients of production. One sees that at once, if, in the definition of such a function, $\Phi_i(nx_{i1}, \cdots, nx_{ir}) = n\Phi_i$, n is put equal to $1/o_i = 1/\Phi_i$, which leads to $1 = \Phi_i(a_{i1}, \cdots, a_{ir})$; except for the additive constant 1, the functional relationship between the coefficients is identical with the production function.

$$0 = \Phi_i(a_{i1}, \cdots, a_{ir}; x_{i1}, \cdots, x_{ir}). \tag{6'}$$

The most serious defect of the set (3a) is the fact that it is applicable only to independent production; in the case of joint production, the simple adding up of the productive services utilized in the production of the different goods will lead to double counting, and a similar difficulty exists for Walras' cost equation,

$$p_i = \sum_{j=1}^{r} a_{ij} q_j, \tag{2a}$$

(where p and q denote the prices of goods produced and productive services respectively). The same difficulty, however, exists for the Paretian system. Both systems represent a simplified approach, which may be accepted in the further discussion of their features.

While, under these simplified conditions, the Walrasian set (3a) is perfectly valid, grave difficulties seem, at first blush, to beset the Paretian set (3b). Pareto introduces it with the following explanation: "The technical conditions of production inform us about the quantities $F_1, F_2, \cdots$ as functions of $o_1, o_2, \cdots$"[5] If this is taken literally, namely as applicable also to the substitutional factors of production, clearly the fundamental question of the choice of the most economical technique would be begged. For in the F_j functions no independent variable denoting this choice appears; the functions contain only the output as variable, despite the fact that the amount of any factor X_j, used up in producing a specified set[6] of produced commodities $o_1', \cdots, o_n'$ is not uniquely associated with these o' values but varies according to the technique applied. Only for the limitational type of coefficients are we informed by the "technical conditions" about the form of the F_j functions and of the $\bar{a}_{ij}$ functions. Moreover, *if* this form should be generally given by the technical conditions, then the system (3a) would be overdetermined in case r is larger than n; in other words, if the amount of productive services per unit of output is technically fixed, then it is not always possible to find a combination of goods in the production of which the available supply of services would be used up.

A literal interpretation of Pareto's sentence, quoted above, is, therefore, inadmissible. It can be understood to refer to our technical knowledge, which enables economic man to select from the whole range of possibilities a particular set of compensatory coefficients, according to a mathematical pattern to be discussed below; integration of these co-

[5] *Manuel*, Appendix, §§70, p. 607. Pareto's notation is replaced by the notation of this paper.

[6] Prime signs, etc., always denote specific values of a variable.

efficients yields the appropriate F_j functions, the form of which is, therefore, an unknown of the problem.[7] The coefficients of production "may be directly given or may be derived from certain systems of equations."[8] From this angle it is easier to understand Pareto's statement that any coefficient is to be considered as a function of only the specific o_i to the production of which it refers.[9] In general, the actual values of any compensatory coefficient depend also upon the input of other services. However, this input is given through the choice of technique, and, therefore, in Pareto's approach, already specified by the form of the function. Pareto's remark is intended to exclude as variables the other $o_1, \cdots, o_h, o_j, \cdots, o_n$ values. In this way, the integrability of the total differential, $dX_j = \sum_{i=1}^{n} \bar{a}_{ij} do_i$, is secured and the F_j function is defined.[10]

It may be noted that this function has an additive character:

$$F_j = \psi_{1j}(o_1) + \psi_{2j}(o_2) + \cdots + \psi_{nj}(o_n).$$

The individual terms on the right-hand side must be identical with the corresponding terms in the Walrasian (3a), on account of the mutual independence:

$$\psi_{ij}(o_i) = a_{ij} o_i, \quad \text{or} \quad X_j = \sum_{i=1}^{n} x_{ij}.$$

Hence the difference between the Walrasian and the Paretian approach lies exclusively in the way in which the various input magnitudes are determined in set (4). We turn to the discussion of this problem.

II

In the Walrasian system, the choice of technique is characterized with the help of the production function, the explicit introduction of which by Walras' immediate successors[11] ultimately rendered the

[7] The present writer, for a long time, has been misled by the questionable sentence. He is indebted to one of the referees who read the MS of this paper for having drawn his attention to the presumably more correct interpretation.

[8] Vilfredo Pareto, "Économie mathématique," §10, in *Encyclopédie des sciences mathématiques*, Paris, 1911.

[9] *Manuel*, Appendix, §78, *Encyclopédie*, §43.

[10] The condition is sufficient but not necessary.

[11] Barone's introduction of the production function was largely accepted by Walras (see *Éléments*, 3rd ed., 1896, Appendix III, "Note sur la réfutation de la théorie Anglaise de fermage de Mr. Wicksteed"). The presentation in the later editions of the *Éléments* is so abridged that Schultz (*loc. cit.*, Appendix II, p. 545) was at a loss to understand why Walras denoted the compensatory relation (6a) and the production function by the same symbol and name. The explanation seems to be that (6a) is only an introductory formulation designed to

coefficients of production and the set (6a) superfluous. Paretians pointed out two difficulties (a) the restriction to homogeneous production functions of the first degree, already mentioned above, and (b) the existence of limitational factors, which impair the general existence of the production function. The first difficulty is not removed by the redefinition of relation (6a), as suggested above; for it arises also for the fact that only a homogeneous function of the first degree *generally* secures equality between the firm's revenue and the payments to the factors of production according to their marginal productivity; in other words, in order that (2a) may be secured for any value of input and output, we need a function for which

$$\Phi_i = \sum_{j=1}^{r} x_{ij} \frac{\partial \Phi_i}{\partial x_{ij}} \cdot \tag{4a'}$$

The homogeneous function of the first degree alone has this property. The difficulty (b) has been mentioned above in connection with the Paretian coefficients: raw materials are the most common (though by no means the only) example of a productive service the role of which in the production process cannot be taken over by any other productive service, so that an isolated increase of the latter would not enhance output, in contrast to the fundamental assumption inherent in the production function, according to which any value of any productive service, other things being equal, is *uniquely* associated with a certain value of o_i.

Both difficulties, however, have been overcome within the framework of the Walrasian system. Utilizing the fact that a nonhomogeneous function might have a *point* (or several) for which (4a') is valid (as an equation, not as a functional relationship), Walras[12] and Wicksell[13] showed that, to reach this point, the mere tendency of the entry of new firms into the industry to make firms of optimum size would be enough.[14] This result remains, of course, indispensable also for the Paretian approach, as revealing the mechanism by which equality between costs and revenue is achieved.

As to point (b), Georgescu-Roegen[15] has shown, more recently, how we can define the production function despite the presence of limita-

set forth the principle of substitution, while the form in which it can be applied definitely is of more general character, and is given by the production function in §326 (edition of 1926).

[12] In the "Note" quoted above.

[13] *Lectures on Political Economy*, 1934, Vol. I, pp. 129–130.

[14] Cf. also J. R. Hicks, *The Theory of Wages*, 1932, p. 237.

[15] N. Georgescu-Roegen, "Fixed Coefficients of Production and Marginal Productivity Theory," *Review of Economic Studies*, Vol. 3, No. 1, October, 1935.

tional factors: while, in absence of limitational factors, the producer has the choice between all the points of the $(r+1)$-dimensional hyperspace, which is defined by the Φ_i function, the choice is narrowed down, in the presence of h limitational factors, to a hypersurface of $(r-h+1)$ dimensions.

This solution of an old problem, however, does not relieve us from the task of investigating whether Pareto's approach, taken by itself, is unobjectionable. We still have to find out, in what way Pareto's fundamental F functions and coefficients are established.

III

On the basis of *hypothetically* given $\bar{a}_{ij}$ functions, Pareto's concept of the total costs of production can be defined in three steps:

(1) Multiplying the coefficients by the prices of the services, we obtain the marginal specific costs of production;

$$\Pi_{ij}=q_j\bar{a}_{ij} \tag{4b.1}$$

(2) Adding up the marginal specific costs of a specific output o_i, we obtain the marginal costs:

$$\Pi_i do_i = do_i \sum_{j=1}^{r} q_j\bar{a}_{ij}. \tag{4b.2}$$

(3) Integrating the marginal costs and taking account of the integration constant (=costs at zero production), we obtain the total costs of production:

$$\Pi_i = C_i + \int_0^{o_i} do_i \sum_{j=1}^{r} q_j\bar{a}_{ij}. \tag{4b.3}$$

Pareto discusses the possibility that the n expressions (4b.2) are partial derivatives of one and the same function of $o_1, \cdots, o_n$, in order to make sure that "the order, the disposition of production" is irrelevant. It is doubtful (and not explicitly stated by Pareto) that the integrability of the whole set (4b.2) is a *necessary* condition. In any case, it should be clear that the transition from (4b.2) to (4.b.3), i.e., the integrability of any particular (4b.2) is always given, whether the $\bar{a}_{ij}$ are functions of one or of several variables, since $do_i\sum_{j=1}^{r} q_j\bar{a}_{ij}$ is not a total differential but has to be integrated with respect to the single variable o_i.

The novelty in Pareto's treatment of the choice of technique consists in varying the *form* of the $\bar{a}_{ij}$ function in (4b.3) in order to find that set $\bar{a}_{ij}$ which gives the minimum value of Π_i. For this purpose the

notation of the calculus of variations is introduced.[16]

$$(4b.4) \qquad 0 = \delta\Pi_i = \int_0^{o_i} do_i(q_1\delta\bar{a}_{i1} + q_2\delta\bar{a}_{i2} + \cdots + q_m\delta\bar{a}_{im}),$$

subject to the condition (6a) $0=\bar{\phi}_i(\bar{a}_{i1}, \bar{a}_{i2}, \cdots, \bar{a}_{im})$

Let us stop here for a moment and examine the question whether the calculus of variations can perform what it is supposed by Pareto to do. Certainly, the conditions (4b.4) and (6b) cannot be sufficient, without any additional information, to select from the infinitely large realm of functions that particular group which, in reality, would minimize costs. In other words, if we want to find that function on which the way between two given points is shortest, then the mathematical expression for "way along a function," together with the minimum condition, is strong enough to exclude from the vast realm of functions all that are not straight lines; in the present case, on the other hand, the mathematical conditions are much too weak to perform a similar task, as any application of the calculus would show.[17]

From the economic angle, the problem looks quite different: the entrepreneur has not his choice between all possible compensatory functions but only between a small number, which are specified by the technical peculiarities of production. This narrower group of admissible functions could be characterized by m parameters, corresponding to the m production factors. We come back to the point later. Here it suffices to point out that if the choice is restricted in advance to a family of functions characterized by a *finite* number of parameters, the ordinary methods of maximizing are entirely adequate, indeed the only applicable ones.

This view is confirmed if we analyze the way Pareto goes on: From (6b), one of the coefficients can be considered as function of the others, which generates the equation:

$$0 = \int_0^{o_i} do_i \left(\left[q_1 \frac{\partial \bar{a}_{i1}}{\partial \bar{a}_{i2}} + q_2 \right] \delta\bar{a}_{i2} + \left[q_1 \frac{\partial \bar{a}_{i1}}{\partial \bar{a}_{i3}} + q_3 \right] \delta\bar{a}_{i3} + \cdots + \left[q_1 \frac{\partial \bar{a}_{i1}}{\partial \bar{a}_{im}} + q_m \right] \delta\bar{a}_{im} \right).$$

Since the variations $\delta\bar{a}_{i1}, \delta\bar{a}_{i2}, \cdots, \delta\bar{a}_{im}$ are independent and quite ar-

[16] $m = m(i)$ denotes the number of compensatory coefficients in the production of good i.

[17] There is an additional condition implied in (4b.4): the $q_j\bar{a}_{ij}$ values cannot be negative and cannot all become zero at the same time. The condition restricts further the range of admissible functions; if the present writer sees correctly, however, the classical calculus is unable to deal with it.

bitrary, this equality can be satisfied identically only if each bracket is zero; this yields, in addition to n equations (6b), $n(m-1)$ equations of the type:

$$q_1 \frac{\partial \bar{a}_{i1}}{\partial \bar{a}_{ij}} + q_j = 0 \qquad (i = 1, 2, \cdots, n; j = 2, 3, \cdots, m).$$

From (6b), the partial differential coefficients $\partial \bar{a}_{i1}/\partial \bar{a}_{ij}$ can be expressed in terms of the $\bar{a}_{ij}$ ($_j$=2, 3, $\cdots$, m), say,

$$\text{(4b.5)} \qquad X_{ij}(\bar{a}_{i2}, \cdots, \bar{a}_{im}) = -\frac{q_j}{q_1},$$

where the X_{ij} are known functions.

But what is really achieved by this procedure? From (4b.5) not the form of the $\bar{a}_{ij}$ functions can be derived but only their equilibrium value in terms of the q's; surely there is an infinitely great number of sets of $\bar{a}_{ij}$ functions which would satisfy (4b.5). On the other hand, in (2b) and (3b), we need the form of the function, and not only their equilibrium value, because otherwise the integration cannot be performed.

$$\text{(2b)} \qquad p_i o_i = C_i + \int_0^{o_i} do_i \sum_{j=1}^{r} q_j \bar{a}_{ij}(o_i),$$

$$\text{(3b)} \qquad X_j = \sum_{i=1}^{n} \int_0^{o_i} \bar{a}_{ij}(o_i) do_i.$$

Hence the Pareto system, consisting of sets (1b), (2b), (3b), (4b.5) and (6b), is not complete. No wonder; for his basis— the demand functions, the X_j magnitudes (or their supply functions), and the $\bar{\phi}$ functions— is extremely scanty.[18] The way out is to embody the existing technical knowledge explicitly in the coefficients. As pointed out above, it is always possible to characterize the technical choice by a finite number of parameters since the number of production factors to be combined is finite; in other words, the entrepreneurs know the limited number of specific input combinations which can produce a given output. Any compensatory coefficient $\bar{a}_{ij}$ is, therefore, a known function of the output o_i and of the other input magnitudes $x_{i1}, \cdots, x_{ii}, x_{ik}, \cdots, x_{im}$. In the form of the functions and the number of variables included the fact of substitutability is already expressed. The

[18] It is remarkable that, in the *Encyclopédie* article, Pareto did not reproduce the calculus-of-variations approach, though he still treated his coefficients as functions of one variable.

integration of the right-hand terms in (2b) and (3b) evidently is not endangered by the new definition of the coefficients. In analogy to the ϕ and $\bar{\phi}$ functions, there is a set,

$$x_{i1} = x_{i1}(x_{i2}, \cdots, x_{im}, o_i), \tag{6c}$$

expressing the fact that for a given output, only $m-1$ compensatory coefficients can be selected arbitrarily.[19] Any set $x_{i1}, \cdots, x_{im}$ represents a specific technique.

The cost expression (4b.3) can then be conceived as a known function of the output variables and of $m-1$ input variables. Differentiating these functions with respect to the input variables yields $n(m-1)$ conditions for the n cost minima, which characterize the equilibrium choice of technique.

The significant idea in Pareto's approach is, therefore, the minimum-cost principle, which permits a satisfactory treatment of the limitational factors. The replacement of the Walrasian coefficients (average magnitudes) by marginal ones is symptomatic but of secondary importance, because of the relation between the two magnitudes:[20]

$$\bar{a}_{ij} = a_{ij} + o_i \frac{\partial a_{ij}}{\partial o_i}.$$

For constant coefficients the second term on the right-hand side vanishes; for coefficients varying proportionately to output ($=c_{ij}o_i$) we have: $a_{ij} = \frac{1}{2}C_{ij}o_i + C_i/o_i$, where C_i is an integration constant. Indeed, we can derive the minimum-cost conditions as well from Walras' set (2a); or even more simply, we can drop the production coefficients entirely and write (2) in the form:

$$p_i o_i = \sum_{j=1}^{r} q_j x_{ij}, \tag{2c}$$

where x_{i1}, according to (6c), is a function of o_i and $x_{i2}, \cdots, x_{im}$. Differentiating with respect to the independent input magnitudes yields a set of $n(m-1)$ equations

$$\frac{\partial x_{i1}}{\partial x_{ij}} = -\frac{q_j}{q_1} \qquad (j = 2, 3, \cdots, m). \tag{4d}$$

[19] It must not be concluded from (6c) that o_i is a function of the $x_{i1}, \cdots, x_{im}$ magnitudes alone.

[20] From $x_{ij} = a_{ij}o_i = C + \int_0^{o_i} \bar{a}_{ij}do_i$.

That is, indeed, Columbus' egg. We have to replace the production function, which disregards limitational factors, by a substitutional function for truly compensatory factors, analogous to the Walrasian (6a). Together with sets (1), (2b), (3) (in the form $X_j=\sum_{i=1}^{n} x_{ij}$), and (6c), the new set (4d) determines the unknowns: prices, output, input.

University of Pennsylvania

CONDITIONS FOR DEMAND CURVES WHOSE CURVES OF TOTAL REVENUE, CONSUMERS' SURPLUS, TOTAL BENEFIT, AND COMPROMISE BENEFIT ARE CONVEX*

By Adolf Kozlik

I. THE TOTAL-REVENUE CURVE

It is important for problems of storage and of price discrimination to know whether the total-revenue curve is convex or concave. I shall try to state the conditions for the demand curves under which the total-revenue curve is convex or concave.

Let D be the quantity. $F(D)$ is the demand curve and $DF(D)$ the total-revenue curve. The total-revenue curve is convex if $d^2[DF(D)]/dD^2<0$; it is a straight line if $d^2[DF(D)]/dD^2=0$; and it is concave if $d^2[DF(D)]/dD^2>0$. In the first case, $d^2[DF(D)]/dD^2<0$ means that $d[DF(D)]/dD$, i.e., the marginal-revenue curve, is falling. If the total-revenue curve is convex, the marginal-revenue curve must therefore be falling. It is generally assumed in the literature (see, for example, Mrs. Robinson's *Economics of Imperfect Competition*, where she deals only with falling marginal-revenue curves) that the marginal-revenue curve is falling. But cases of increasing marginal-revenue curves can easily occur.

Since $d^2[DF(D)]/dD^2=DF''(D)+2F'(D)$, the condition for a convex, straight-line, or concave total-revenue curve is $DF''(D)+2F'(D)\gtreqless 0$, or $DF''(D)\gtreqless -2F'(D)$.

If the total-revenue curve is a straight line, $DF''(D)=-2F'(D)$, and by solving this differential equation we get $F(D)=a/D+b$, where a and b are constants ($a>0$). These are Marshall's constant-outlay curves, a/D, shifted up or down by any constant quantity b (Figure 2).

If the case of a straight-line total-revenue curve, $DF(D)=a+bD$, we have $F'(D)=-a/D^2$ (Figures 1, 2), $F''(D)=2a/D^3$ (Figure 1), and $DF''(D)=2a/D^2$ (Figure 1). In Figure 2 some demand curves and their marginal curves for straight-line total-revenue curves are represented.

If the marginal curve, $F'(D)=I'(D)$, of a given demand curve, $F(D)=I(D)$, cuts one of the curves $F'(D)=-a/D^2$ from above at a point P', then its slope at this point is smaller than the slope of the curve $F'(D)=-a/D^2$.

Since the slope of the curve $F'(D)=I'(D)$ is $F''(D)=I''(D)$ and the slope of the curve $F'(D)=-a/D^2$ is $F''(D)=2a/D^3$, the relation

* Journal paper No. J-765 of the Iowa Agricultural Experiment Station, Ames, Iowa. Project No. 383.

$I''(D)<2a/D^3$, or $DI''(D)<2a/D^2$ (that is, $AB<AP''$ in Figure 3) must hold true. Now the curves $F'(D)=I'(D)$ and $F'(D)=-a/D^2$ intersect each other at P', and hence $I'(D)=-a/D^2$ at P'. From this and from $DI''(D)<2a/D^2$, it follows that $DI''(D)<-2I'(D)$. Thus $AB<AP''$ and $AP''=2AP'$, and therefore $AB<2AP'$. This, as was shown above, is the condition for a convex total-revenue curve $I(D)$. It means: The total-revenue curve is convex if the marginal-demand curve cuts the curve $F(D)=-a/D^2$ from above.

If the derivative $F'(D)=I'(D)$ cuts one of the curves $F'(D)=-a/D^2$ from above at P' (Figure 4), then the following relations hold true:

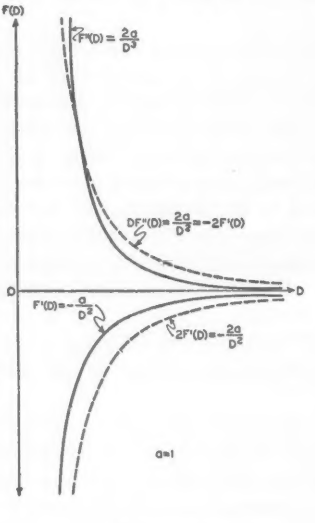

FIGURE 1.

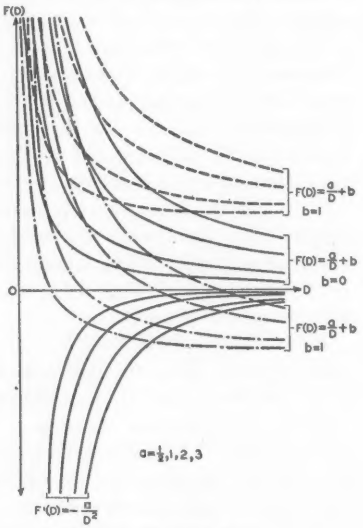

FIGURE 2.

at P', $I'(D)=-a/D^2$; at the left of P', $I'(D)>-a/D^2$; and at the right of P', $I'(D)<-a/D^2$. The curve $F(D)=I(D)$ must therefore have the same slope at P as any one of the curves $F(D)=a/D+b$; at the left of P it must be more level than these curves; and at the right of P it must be steeper than these curves. This means that the curve $F(D)=I(D)$ must be tangent from below to the curve $F(D)=a/D+b$, and less concave than that curve.

Therefore: The total-revenue curve is convex if the demand curve is less concave than the curve $F(D)=a/D+b$, which is tangent to it at this point.

A similar statement holds true for a concave total-revenue curve: If the marginal curve, $F'(D)=\Pi'(D)$, of a given demand curve, $F(D)$

$=\Pi(D)$, cuts one of the curves, $F'(D)=-a/D^2$, from below at a point P' (Figure 3), then its slope at that point is greater than the slope of the curve $F'(D)=-a/D^2$, and $\Pi''(D)>2a/D^3$ or $D\Pi''(D)>2a/D^2$ (that is, $AC>AP''$) and hence $D\Pi''(D)>-2\Pi'(D)$. Thus $AC>AP''$ and $AP''=2AP'$, and therefore $AC>2AP'$. This is the condition for a concave total-revenue curve, $\Pi(D)$. The total-revenue curve is concave if the marginal-demand curve cuts the curve $F'(D)=-a/D^2$ from below. In this case the following relation holds true (Figure 4): at P', $\Pi'(D)=-a/D^2$; at the left of P', $\Pi''(D)<-a/D^2$;

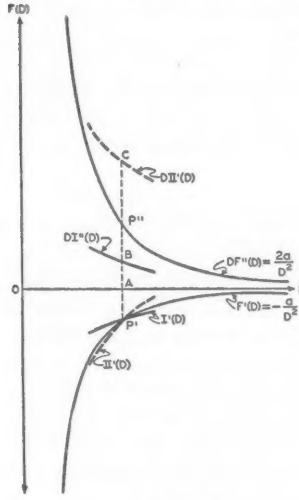

FIGURE 3.

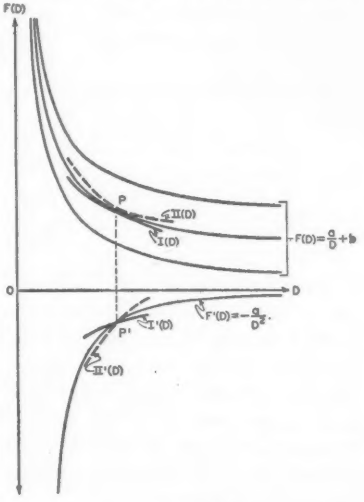

FIGURE 4.

and at the right of P', $\Pi''(D)>-a/D^2$. The curve $F(D)=\Pi(D)$ must therefore have the same slope at P as any of the curves $F(D)=a/D+b$; at the left of P it must be steeper than these curves; and at the right of P it must be more level than these curves. This means that the curve $F(D)=\Pi(D)$ must be tangent from above to the curve $F(D)=a/D+b$, and must be more concave than that curve.

Therefore: If the demand curve is more concave than the curve $F(D)=a/D+b$ which is tangent to it at this point, the total-revenue curve is concave.

Since the curves $F(D)=a/D+b$ are concave, a straight-line or convex demand curve is always less concave than such a curve, and the total-revenue curve of a straight-line or convex demand curve is always convex. If $F''(D)\leqq 0$ (convex- or straight-line demand curve) and $F'(D)<0$ (falling demand curve), the condition for a convex

total-revenue curve, $DF''(D) < -2F'(D)$, is always fulfilled, since the left-hand side of the equation is negative and the right-hand side is positive.

If we draw the demand curve on a logarithmic grid, the slope of the demand curve with respect to the price axis represents the elasticity of demand. If the demand curve on a logarithmic grid is convex, it means that the elasticity of demand, η, decreases with increasing D, that is, $d\eta/dD < 0$. From the decreasing elasticity of the demand curve it does not follow that the total-revenue curve is convex:

$$\eta = -\frac{dD}{dF(D)}\frac{F(D)}{D}.$$

If $d\eta/dD < 0$ (convex demand curve on logarithmic grid), we have:

$$\frac{d\eta}{dD} = \frac{-[F'(D)]^2 D - F(D)[F''(D)D + F'(D)]}{[F'(D)]^2 D^2} < 0, \quad \text{or}$$

$$[DF''(D) + 2F'(D)] - \frac{F'(D)}{F(D)}[DF'(D) + F(D)] < 0.$$

If the marginal revenue $DF'(D)+F(D) > 0$, that is, if the total-revenue curve is rising, then the second term of the last equation above is greater than 0, since $F(D) > 0$ and $F'(D) < 0$ (falling demand curve), and then the first term of the same equation must be less than 0, and the total-revenue curve is convex. If the marginal revenue is negative, that means that, when the total-revenue curve is falling, the total-revenue curve can be convex, concave, or a straight line, even if the demand curve is convex on a logarithmic grid. Another proof for this is given by Allen.[1]

If the demand curve on a logarithmic grid is convex, i.e., if the elasticity of demand is decreasing, then the total-revenue curve must be convex in its *rising* part, but one cannot make the same conclusion as to the *falling* part. The conclusions of Professor Shepherd[2] hold true only for the rising part of the total-revenue curve, for demand curves with elasticity $\eta > 1$.

II. THE CURVES OF TOTAL REVENUE, CONSUMERS' SURPLUS, TOTAL BENEFIT, AND COMPROMISE BENEFIT

The relation of the total-revenue curve, the consumers'-surplus curve, the total-benefit curve, and the compromise-benefit curve

[1] R. G. D. Allen, *Mathematical Analysis for Economists*, pp. 257 f.

[2] Geoffrey Shepherd, "Price Discrimination for Agricultural Products," *Journal of Farm Economics*, Vol. 20, pp. 792 ff.

(Marshall) is also important for the problem of storage. I shall deal with this problem: Under what conditions is each of these curves convex or concave and to what degree are these conditions compatible?

The demand curve is $p=F(D)$. The total-revenue curve is $DF(D)$, the total-benefit curve is $\int_0^D F(D)dD$, the consumers'-surplus curve is $\int_0^D F(D)dD-DF(D)$, and the compromise-benefit curve is $\int_0^D F(D)dD-DF(D)+nDF(D)=\int_0^D F(D)dD-(1-n)DF(D)$, where n indicates how many units of consumers' surplus are considered equal in value to one unit of total revenue.

Since we know only the demand curve, if we know anything at all, and not the exact curve of the consumers' surplus per unit of the good, we neglect the difference between the curves.[3] If we assume that the difference increases proportionally or at all with D—which seems to be very likely—then the difference does not have much influence on the shape of the benefit curves or of the consumers'-surplus curve.

The conditions for the convexity of the four curves are:

Total-revenue curve, $DF(D)$: convex if $DF''(D)+2F'(D)<0$.

Total-benefit curve, $\int_0^D F(D)dD$: convex if $F'(D)<0$ or $DF''(D)+2F'(D)<DF''(D)+F'(D)$.

Consumers'-surplus curve, $\int_0^D F(D)dD-DF(D)$: convex if $-[DF''(D)+F'(D)]<0$ or $DF''(D)+2F'(D)>F'(D)$.

Compromise-benefit curve, $\int_0^D F(D)dD-(1-n)DF(D)$: convex if

$$F'(D)-(1-n)[DF''(D)+2F'(D)]<0, \quad \text{or}$$

$$\begin{cases} DF''(D)+2F'(D) > \dfrac{F'(D)}{1-n} & (n<1), \\ DF''(D)+2F'(D) < \dfrac{F'(D)}{1-n} & (n>1), \\ DF''(D)+2F'(D) < DF''(D)+F'(D) & (n=1). \end{cases}$$

Since the demand curve is assumed to be falling, $F'(D)<0$. This is the condition for a convex total-benefit curve, as was just shown, and the total-benefit curve, $\int_0^D F(D)dD$, therefore is always convex. The compromise-benefit curve coincides with the total-benefit curve if $n=1$; the compromise-benefit curve coincides with the total-revenue curve if $n=\infty$; and the compromise-benefit curve coincides with the consumers'-surplus curve if $n=0$.

If $n>1$, then $F'(D)/(1-n)>0$ and, if $DF''(D)+2F'(D)<0$ (convex total-revenue curve), then $DF''(D)+2F'(D)<F'(D)/(1-n)$ (convex compromise-benefit curve). If $n=1$, then the compromise-benefit curve is convex, because $F'(D)<0$. This means: The compromise-benefit

[3] See J. R. Hicks, *Value and Capital*, pp. 39 ff.

curve is convex if the total-revenue curve is convex and if one unit of consumers' surplus is not valued more than one unit of total revenue. The compromise-benefit curve is always convex if one unit of total revenue is valued the same as one unit of consumers' surplus, i.e., if the compromise-benefit curve becomes the total-benefit curve. The total-benefit curve, therefore, is always convex.

If $n<1$, then both the total-revenue curve and the compromise-benefit curve are convex, provided

$$DF''(D)+2F'(D)< \quad 0 \quad \text{(convex total-revenue curve) and}$$

$$DF''(D)+2F'(D)>\frac{F'(D)}{1-n} \quad \text{(convex compromise-benefit curve), or if}$$

$$-2F'(D)>DF''(D)>F'(D)\,\frac{2n-1}{1-n}\,.$$

Since in the limit, if $n=0$ (consumers'-surplus curve), $(2n-1)/(1-n)$ becomes -1, and since for $1>n>0$ we have $(2n-1)/(1-n)>-1$, both inequalities are compatible, for $-2F'(D)>F'(D)(2n-1)/(1-n)$ if $(2n-1)/(1-n)>-2$, which holds true for $1>n>0$. But there are only a few demand curves which fulfil this condition if n becomes considerably smaller than 1.

If the compromise-benefit curve is a straight line, then $DF''(D)=F'(D)(2n-1)/(1-n)$, and from this we get by solving the equation:

$$F(D) = a\,\frac{1-n}{2n-1}\,D^{n/(1-n)} + b$$

$$\left(a\,\frac{2n-1}{n} < 0 \text{ if } 0<n<1,\ a\,\frac{2n-1}{n} > 0 \text{ if } n>1\right),$$

$$F'(D) = aD^{(2n-1)/(1-n)},$$

$$F''(D) = a\,\frac{2n-1}{1-n}\,D^{(3n-2)/(1-n)}.$$

In the limit if $n=0$ (consumers'-surplus curve), then $DF''(D)=-F'(D)$. From this we get by solving the equation:

$$F(D) = a\log D + b,$$

$$F'(D) = \frac{a}{D},$$

$$F''(D) = -\frac{a}{D^2} \qquad (a<0).$$

As in the case of the total-revenue curve, the consumers'-surplus curve is convex if the demand curve is more concave than the one of the curves $F(D)=a \log D+b$ which is tangent to the demand curve at this point. If the demand curve is less concave than this curve, then the consumers'-surplus curve is concave.

In Figure 5 it is shown that $DI''(D) > -I'(D)$ (condition for a convex consumers'-surplus curve), if $I'(D)$ cuts the curve $F'(D)=a/D$ from below. This is the case, as is shown in Figure 2, if demand curve

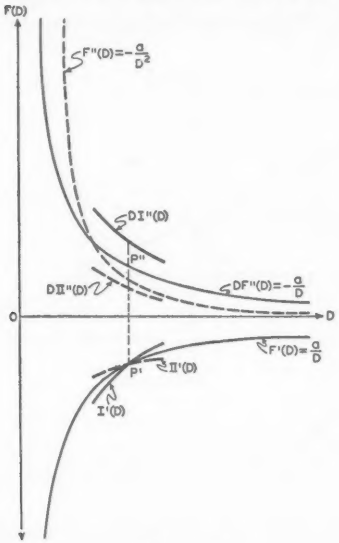

FIGURE 5.

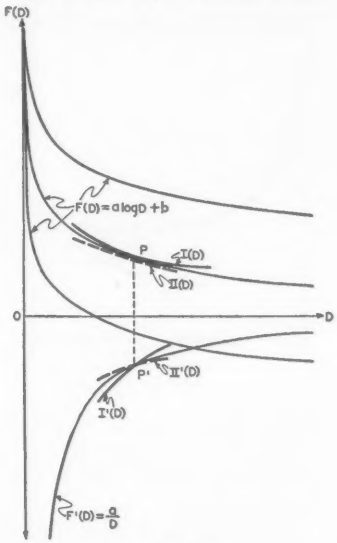

FIGURE 6.

$I(D)$ is tangent to curve $F(D)=a \log D+b$ from above; that is, if the demand curve $I(D)$ is more concave than the curve $F(D)=a \log D+b$.

The contrary holds true if the demand curve $\Pi(D)$ is less concave than the curve $F(D)=a \log D+b$, i.e., if the demand curve $\Pi(D)$ (Figure 6) is tangent to the curve $F(D)=a \log D+b$ from below. In this case the marginal demand curve, $\Pi'(D)$, cuts the curve $F'(D)=a/D$ from above and (Figure 5) in this case $D\Pi''(D) < -\Pi'(D)$, and the condition for a concave consumers'-surplus curve is fulfilled.

If the elasticity of the demand curve increases with increasing D, then the consumers'-surplus curve is convex. If

$$\frac{d\eta}{dD} = \frac{-[F'(D)]^2 D - [F(D)F''(D)D + F'(D)]}{[F'(D)]^2 D^2} > 0$$

or

$$F''(D)D + F'(D) > \frac{[F'(D)]^2 D}{F(D)},$$

the condition $DF''(D)+F'(D)>0$ for a convex comsumer's-surplus curve is fulfilled, since $[F'(D)]^2D/F(D)$ is positive.

Hence the demand curve, $F(D)$, must be less concave than the one of the curves, $F(D)=a/D+b$, which is tangent to it at this point, and more concave than the one of the curves $F(D)=a \log D+b$ which is tangent to it at this point, if both the total-revenue and the consumers'-

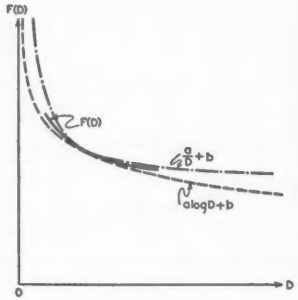

FIGURE 7.

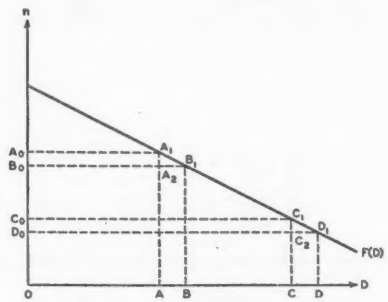

FIGURE 8.

surplus curves are to be convex (Figure 7.) Since the curve $F(D) = a \log D + b$ is concave, a straight-line or convex demand curve is always less concave, and in this case the consumers'-surplus curve is always concave.

This is shown in Figure 8, where $F(D)$ is a straight-line demand curve. If the quantity increases from OA by AB, the consumers' surplus increases by $A_0B_0B_1A_1$; if the quantity increases from OC by the same amount $CD(=AB)$, then the consumers' surplus increases by $C_0D_0D_1C_1$.

Since $A_1A_2B_1 \equiv C_1C_2D_1$, $C_0D_0 \equiv A_0B_0$ (if the demand curve is convex, $C_0D_0 > A_0B_0$) and $C_0D_0D_1C_1 > A_0B_0B_1A_1$. The increase of consumers' surplus at C is greater than at A; the consumers'-surplus curve is steeper at C than at A, i.e., it is concave.

If n varies from 0 to 1, the curve

$$F(D) = a\frac{2n-1}{n} D^{n/(1-n)} + b \qquad \left(a\frac{2n-1}{n} < 0\right)$$

(demand curve of a straight-line consumers'-surplus curve) becomes less and less concave. If $n=\frac{1}{2}$, the curve becomes a straight line,

$F(D)=cD+b$. If $n>\frac{1}{2}$, the curve becomes convex until it reaches the greatest possible convexity at $n=1$, and every demand curve is less convex than the curve

$$F(D) = a\frac{2n-1}{n}D^{n/(n-1)} + b \qquad (n = 1,\ a < 0).$$

Figure 9 shows how the curve

$$F(D) = a\frac{2n-1}{n}D^{n/(1-n)} + b \qquad \left(a\frac{2n-1}{n} < 0\right)$$

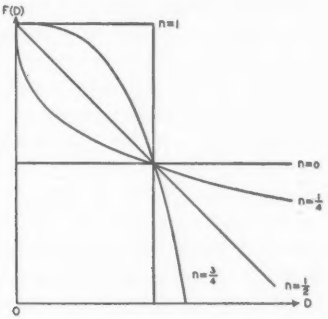

FIGURE 9.

becomes less and less concave and then more and more convex as n increases from 0 to 1. The probability that both the total-revenue curve and the compromise-benefit curve are convex increases therefore as n goes from 0 to 1. The more nearly a unit of the consumers' surplus is equated to one unit of the total revenue, the more convex becomes the compromise-benefit curve; and the less concave the demand curve is, the more convex becomes the total-revenue curve and the less convex becomes the consumers'-surplus curve, whereas the total-benefit curve is in any case convex.

All curves can be convex at the same time only in the case of a few special forms of demand curves, but the total-revenue curve, the total-benefit curve, and the compromise-benefit curve can easily be convex together.

Iowa State College

INTEREST RATES: LONG-TERM vs. SHORT-TERM

By J. Leland Daniel

Everyone is vitally interested in the trend of interest rates, and particularly as to how soon a definite rise in rates may take place. On this question hinge the welfare of our banking institutions, the value of bond portfolios held in various types of trust funds, the future net premium rates on life insurance policies, the cost of carrying the national debt and, to a certain extent, the distribution of the country's wealth. Consequently, every investor would like to protect himself against drastic changes in interest levels. If he is to accomplish this he must be able to recognize a movement in its early stages, or better still, to forecast the turning points in an interest cycle.

Let us take up the relationship of long- and short-term interest rates, especially the relationship of their important movements. This relationship presents some very interesting questions. Do long-term and short-term interest rates move together? Does a directional shift of either presage a prompt change in trend of the other? Can the two properly be considered a single factor? Or, on the other hand, are long-term and short-term rates two separate and distinct elements, each having its own cycle pattern which is entirely independent of the other?

While some economists recognize a difference between long- and short-term interest rates, many writers on economic and financial matters are wont to generalize about interest rates and to make no distinction as to term. Those who do make a distinction are prone to pass over the subject lightly;[1] and the usual conclusion is that the two classes of rates tend to move together.

Believing the subject to be of prime importance from an investment standpoint, the author has been engaged in an historical survey of the problem.

By "interest rate" herein is meant as near the pure or riskless rate as practicable. This means taking the highest grade of bonds, preferably of ten years or greater maturity where data are available, for long-term rates, and prime commercial paper, bankers' bills, etc., for short-term rates. To consider anything of lesser quality than these would introduce the risk factor to such an extent that the results of the study would be misleading as a guide to movements of interest rates.

Inasmuch as this study is concerned primarily with turning points of *major* cycles of interest rates, the intermediate swings are to be disregarded. The trends which did not continue for approximately a year or longer will be eliminated.

[1] However, in recent years the works of Frederick R. Macaulay and Ralph G. G. Hawtrey are notable exceptions.

Both English and American interest rates were surveyed. Interest rates in England were traced back to 1831. For long-term rates the yield on 2½-per-cent consols were used. The consols bore 3 per cent nominal interest from 1751–1888, and in 1889 were converted into consols bearing 2¾ per cent for fourteen years and 2½ per cent thereafter. The average amount outstanding throughout the period under consideration was in the neighborhood of £400,000,000. They are perpetuities, although since 1923 the British Treasury has been authorized to pay them off at par on three-months notice if it so desires. Such redemption appears entirely unlikely.

In plotting the movement of short-term rates in England several sources were used. Jevons[2] gives market rate of discount "as actually charged at the beginning of each month at the house of Overend, Gurney and Co." to May, 1857, and Bank of England minimum rate of discount thereafter to June, 1883. A. C. Pigou[3] has a chart showing monthly market rates of discount on three-months bank bills in the London market, 1848 to 1912. T. T. Williamson[4] has tabulated the annual rate for good three-months bank bills in the market, 1845 to 1909. Later rates were obtained from *Statistical Abstract for United Kingdom.*

The various series of discount rates correlated very closely, and there was no difficulty in arriving at the turning points from the peaks and troughs of major cycles of short-term rates in England. In some instances a rate would stay at a high or low level for several months before starting a new swing. In such cases the last month before a new rate was established was considered the turning point rather than the one in which the level was reached.

Consol yields were also checked against several sources in order to insure accuracy in fixing the turning points. Close agreement was found among the various sources.

In identifying the major trends of consol yields, swings which did not continue for approximately a year or longer were disregarded; and movements involving less than five points (about 0.15 yield points) were not considered. That is to say, no movement of consol yields was considered a major movement unless it involved at least 5 points and continued approximately a year or longer.

The required amplitude for a major cycle varies somewhat with the security under consideration, because the terms "major" and "intermediate" are relative; but for the most part the major cycles were easily discernible by the pattern of the curve. There were only a few

[2] *Investigations in Currency and Finance*, London, 1884.
[3] *Industrial Fluctuations*, London, 1927.
[4] *Journal of the Royal Statistical Society*, March, 1912.

instances where there was some question as to whether a movement constituted a reversal of the major trend or was merely part of an intermediate cycle.

The two English series were plotted on the upper part of the accompanying Figure 1. Let us examine them for any important relationship between the consol curve (long-term rates) and the curve of short-term rates. We are here particularly interested in (1) whether the two curves move together, and, what is more important, (2) whether the movement of one invariably forecasts movement of the other.

This chart does not purport to show intermediate movements. The curves have been smoothed by drawing a straight line connecting the turning points in major interest cycles. By thus eliminating the minor swings a comparison of major cycles is more easily made, and the turning points are emphasized. As previously stated, the swings of the minor cycles are relatively unimportant.

The chart illustrates two important statistical factors, (1) the *trend* of interest rates, both long- and short-term, at any given date, and (2) the dates of *reversals of direction* of interest movements.

One should not be misled by the amplitude of the movements on the chart. While the amplitude of a movement was given consideration, along with the length of time it continued, in determining which were major trends, yet some of these movements remained at or near their extremes for relatively brief periods. These extreme points are necessary, however, in determining *when* reversal took place.

It should be noted here that, for short-term rates from 1868 to 1909 there are used the annual average rates of discount on three-months bank bills as charted by A. C. Pigou and tabulated by T. T. Williamson. While the years are probably correct for the ones in which extreme quotations occurred, the actual extremes were probably considerably higher or lower than these annual averages.

Two particularly interesting facts are brought into relief by this chart. First, the remarkable steadfastness of consols for a large part of the period covered, and second the comparative instability of short-term rates. For many years the only important change in consol yields was the gradual secular down trend.

From a price of about 86 (yield 3½ per cent) in June, 1866 the consol market rose swiftly to about 95 in October, 1867. From then on it never dipped again as low as 90 (and never had an intermediate recession from the gradual upswing of more than four points) until after 1897. This was in spite of the wide fluctuations in trade activity; also despite the fact that discount rates showed wide fluctuations.

The gap in short-term rates 1910–1920 is on account of the war period. It will be noted that during the decade beginning with 1920

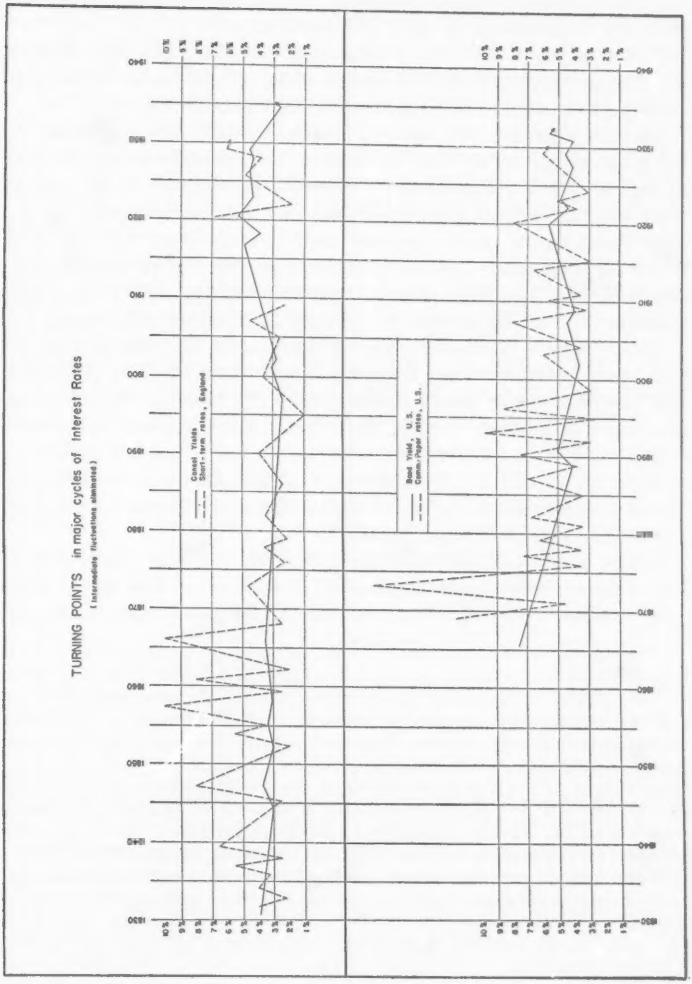
TURNING POINTS in major cycles of Interest Rates
(Intermediate fluctuations eliminated)
Consol Yields
Short-term rates, England
Bond Yield, U.S.
Comm. Paper rates, U.S.

FIGURE 1

there was fairly close correlation between the two series. Both types of interest rates started downward in December, 1920; and the downward movements of short-term rates which began in 1926 and in 1929 preceded long-term rates by a few months.

The upswing of short-term rates beginning in July, 1922 forecast the change in consol trends by nine months, but the rise of bill rates in 1929 began a month after consol yields had started upward.

The short-term curve ends in October, 1929; for the decline in bill rates which began that month has not definitely reversed itself. Except for short seasonal movements this rate (prior to the present war) has remained at about 9/16 per cent for several years. On the other hand consol yields reached their low of about 2.66 per cent (price $94\frac{3}{8}$) in January, 1935, from which point the yield gradually rose, and on August 10, 1939 it was 4.00 per cent and the price around $62\frac{1}{2}$, a decline of about 32 points. On January 23, 1940 the price was at 73.

The inevitable conclusion seems to be that in England *the correlation between short-term and long-term interest rates has been poor.* Not only have long-term rates frequently moved without a corresponding movement of short-term rates, but what is more marked, short-term rates have had many cycles not participated in by long-term rates.

Next, let us turn to the lower part of the chart and examine the curves which indicate the approximate dates of changes in trend of interest rates in the United States.[5]

A mere glance will serve to convince one that in this country, as in England, short-term interest rates have been much less stable than long-term rates; and that rates on commercial paper have had major

[5] The rates for commercial paper in the United States were obtained from a table compiled from various sources by Col. Leonard P. Ayres of the Cleveland Trust Company. To arrive at turning points in major cycles of bond yields in U. S. the chart in the *Annalist* for January 25, 1939 and seven other reliable sources were considered. Some of these sources covered considerably longer periods than others. Where there was disagreement among the sources those turning points were used which were confirmed by the greatest number of sources. In general there was very good correlation. For yields at the turning points those derived from the *Annalist* chart above referred to were used. There was little difficulty in distinguishing between major and intermediate cycles of bond yields. Major movements were all approximately a year or more in length, and generally over $\frac{1}{2}$ per cent in yield and 10 per cent of value. Reactions were usually very brief, never over six months; they were usually less than $\frac{1}{4}$ per cent in yield, never over $\frac{1}{2}$ per cent, usually less than 25 per cent of amplitude of the major movement, never over 50 per cent.

In determining whether a movement is major or intermediate not only the above factors must be taken into consideration, but also the general pattern of curve, which includes the definiteness of the swings immediately before and after the movement being considered, and also the nature of any reactions to the movement under consideration.

cycles, some of great magnitude, which if reflected at all in long-term yields are reflected to a negligible extent. Thus, long- and short-term rates *often move in opposite directions.* A closer inspection of the chart, however, will disclose one interesting similarity in the two curves, namely, that the reversal of trend of bond yields was almost always accompanied or preceded by a turn in short-term rates.

There were only a few scattered years from 1890 to 1926 in which commercial-paper rates did not fluctuate over a range of at least 1 per cent and many years had a range of 2 per cent or more.

We believe the chart illustrates conclusively the nondependability of short-term interest rates as a barometer of long-term interest-rate movements.

No doubt reasons can be advanced for some of the peculiar movements of short-term rates; however, this does not alter the inevitable conclusion that the next reversal of short-term rates will not necessarily augur a similar trend in bond yields.

But an important conclusion to be derived from this study is that while short-term rates may turn upward without a subsequent similar trend of long-term rates, the movements of short-term rates must nevertheless be watched very carefully. For, as previously stated, almost every major movement of long-term rates in this country during the past seventy years has been preceded or accompanied by a similar movement of short-term rates. True, the upturn of short-term rates in 1899 occurred about three years ahead of the upturn of long-term rates; nevertheless, the movement of long-term rates was preceded by short-term rates. In other words, in this country long-term rates have seldom reversed their trend without previously or concurrently a reversal of short-term rates.

This is not an infallible rule, however, for it has been shown that in England the reversal of direction of consol yields was not preceded very consistently by a reversal in short-term rates. While there was a corresponding movement in short-term rates for each movement in consol yields, the upturn of consol yields has been known to precede the upswing of short-term rates by as much as thirteen months.

An interesting exception in the United States was the rise of bond yields which began about July, 1931 and which for purposes of this study was considered a major movement. But commercial-paper rates, while they rose sharply shortly thereafter, resumed their decline about four months later, so theirs was considered only an intermediate movement.

A comparison of the turning points of long-term bond yields in the United States with turning points in consol yields does not disclose very good correlation between the two countries. However, the rever-

sals from peaks of United States bond yields in 1920 and 1929 were followed a few months later by major downswings of consol yields. Also the turning points from low bond yields in the United States in 1902 and 1928 were followed very shortly by a like movement in consol yields. But the instances of nonconformity are in the majority.

To recapitulate, short-term interest rates cannot be depended upon as a barometer of change in direction of major movements of long-term interest rates because short-term rates have so many more cycles than do long-term rates. A negative rule—for current consideration—can be formulated, based upon experience in the United States, to the effect that long-term rates *probably will not* reverse their trend until commercial-paper rates have definitely started upward. That even this rule is subject to important exceptions is shown by present and past price movements of British consols.

Spokane, Washington

REPORT OF THE ELSINORE MEETING, AUGUST 25–26, 1939

By P. DE WOLFF

THE NINTH European meeting of the Econometric Society was planned to be held at Elsinore, Denmark, on August 26–28, 1939. The first day was intended to coincide with the last day of the fourth annual meeting of the Society of Nordic Economists and to be dedicated to subjects of common interest. Both meetings were carefully prepared by Professor Zeuthen from Copenhagen but, unfortunately, on account of the political situation of the moment, most non-Scandinavian members were prevented from attending and the total duration of both meetings was therefore restricted to three days (August 24–26). This, however, had the advantage that nearly all participators in the meeting of the Nordic Economists were present during all the lectures of the Econometric meeting. The following members attended: B. Barfod, Aarhus; Professor A. Bowley, Haslemere; B. Gloerfelt-Tarp, Copenhagen; Dr. Ivar Jantzen, Copenhagen; Dr. Margaret W. Joseph, London; Professor E. Lindahl, Lund; Professor E. Schneider, Aarhus; P. de Wolff, The Hague; Dr. H. Wold, Stockholm; and Professor F. Zeuthen, Copenhagen. Moreover, 14 members of the Society of Nordic Economists were present at one or more of the sessions.

The first session of the Econometric meeting was held in the evening of Friday, August 25, with Professor Zeuthen in the chair. The following papers were delivered:

P. DE WOLFF, The Hague, *A Preliminary Model of the Swedish Trade Cycle.*—A paper on this subject will soon be published in ECONOMETRICA.

B. BARFOD, Aarhus, *The Theory of Advertising.*—In the ordinary theory of monopoly the monopolist has only one "action parameter," viz., price, at his disposition. Mr. Barfod generalized this theory by introducing advertisement. It is evident that, *ceteris paribus*, the carrying out of a well-directed publicity campaign will increase the total volume of sales. Different forms of advertisement can be distinguished:

1. Informing advertisement, giving information about the qualities of the product sold.

2. Suggestive advertisement, suggesting the superiority of the product over similar products.

Each form of advertising can be realized by different means, called "excitation factors" (e.g., newspaper advertising, packaging, etc.). The assumption is made that the different excitation factors can be measured and that the quantity x, sold at a price p, when doses w_1,

$w_2, \cdots$ of the various excitation factors are administered to the purchasers, satisfies a functional relation of the following form:

$$F(x, p, w_1, w_2, \cdots).$$

Or, in the case of only two factors:

$$F(x, p, w_1, w_2) = 0. \tag{1}$$

The advertisement cost R is given by:

$$R = \pi_1 w_1 + \pi_2 w_2, \tag{2}$$

π_1 and π_2 being the unit prices of the two excitation factors. Now the way of advertising yielding the greatest profit will be determined. This problem is solved by purely geometrical methods of the sort generally applied by Professor Zeuthen and his pupils. The solution is performed in two steps. First, with a given R, the ratio in which R has to be divided between both excitation factors is fixed. Second, the most profitable value of R is determined. For a given value of R the ratio $\lambda = w_1/w_2$ may still be chosen at will and to every value of λ corresponds a definite "ordinary" demand curve, obtained from (1) by inserting for w_1 and w_2 the values obtained from (2) and λ. Varying λ we obtain an infinite number of such curves and it is evident that the situation which yields maximum output at a given price will be found on the envelope of this system of curves (in order that this curve in fact leads to a maximum and not to a minimum it is necessary that the individual demand curves turn their concave side to the origin at least in a restricted part of their course). An enveloping curve can be constructed for every value of R, the curves generated in this way are called iso-R-curves. The most profitable situation T on a given iso-R-curve can be determined with the help of another system of curves, the iso-Q-curves, where Q represents profit:

$$Q = px - k(x), \tag{3}$$

where $k(x)$ is total cost at the output x, *excluding* cost of advertising, which, however, is constant at an iso-R-curve. Among the iso-Q-curves there will in general be one touching the given iso-R-curve. If T coincides with this tangent point it will evidently lie on the iso-Q-curve with the highest value of Q. The second step can now easily be taken. When R is changed T will describe a curve, the so-called excitation curve. The situation which yields the absolute maximum of profit (after deduction of the cost of advertisement) will be found on the excitation curve. It will be the point where the marginal rates of increase of Q and R *along the curve* are equal,

During the discussion Mr. Barfod referred to a review of his book

Reklamen i Teoretisk-økonomisk Belysning in *Nordisk Tidskrift for Teknisk økonomi*, June, 1938, by Professor Schneider where an analytical solution of the same problem is given.

At the second session, in the morning of Saturday, August 26, Professor A. L. BOWLEY presided. Papers were delivered as follows:

H. WOLD, Stockholm, *Some Problems of Method in Statistical Investigation of Economic Relations.*—Dr. Wold stated that the chief problem in statistical regression analysis is to determine the true regression equation between a number of variables. Many different solutions have been suggested. They differ in the assumptions made about the character of the errors taken into account, and they lead to elementary, orthogonal, diagonal regression, and so on. It is of paramount importance to distinguish between two different types of errors:

1. *Errors of estimation:* These errors are real and due to other factors not taken into account in the investigation.

2. *Errors of observations:* These errors are due to inaccuracy of the observations.

Only the second class can be corrected for. This was shown in a simple case. The first type, however, introduces a stochastical element, which cannot be eliminated by any ever so complicated method of regression analysis. Dr. Wold illustrated the consequences of his point of view by a classical example. If we plot in a diagram the heights of father and son in a great number of families as abscissae and ordinates we shall find a scatter. In other words, there is not a functional but a stochastical relation. Now there is no sense in looking for a true regression, describing the relation between the two heights, because we must use *two different relations.* If we want to estimate the most probable height of the son corresponding to a given height of the father, we must use the first elementary regression, whereas the solution of the reversed problem is given by the second elementary regression. The same holds true for economic relations, e.g., the relation between expenditure on a certain commodity and income. The presence of observational errors may necessitate the use of regression equations somewhat different from the elementary regressions; nevertheless the main point remains true that different equations must be used to estimate expenditure from income and income from expenditure. The diagonal regression, recommended by Professor Frisch, should be looked upon as describing a structural relation between the two variables but should not be used for the problem of estimation, mentioned above. Denoting the first elementary regression coefficient between two variables by e', the diagonal regression coefficient by e''', and the correlation coefficient by r, the following relation exists:

$$e''' = r^{-1}e'.$$

As $r \leqq 1$ this leads to $e''' \geqq e'$. Therefore, Dr. Wold concludes that the price and income elasticities computed by Professor Frisch, e.g., in his demand study for milk,[1] generally are too high. In conclusion Dr. Wold treated the results of an investigation into income elasticities for a number of commodities. If the expenditure for a certain commodity is denoted by x, the income elasticity E_x is given by:

$$E_x = \frac{I}{x} \frac{\partial x}{\partial I}.$$

The income elasticity E_n for a group of commodities $x_1, x_2, \cdots x_n$ is given by

$$(1) \quad \begin{aligned} E_n &= \frac{I}{x_1 + x_2 + \cdots + x_n} \left[\frac{\partial x_1}{\partial I} + \frac{\partial x_2}{\partial I} + \cdots + \frac{\partial x_n}{\partial I} \right] \\ &= \frac{1}{x_1 + x_2 + \cdots + x_n} [x_1 E_{x_1} + x_2 E_{x_2} + \cdots + x_n E_{x_n}]. \end{aligned}$$

In words: the income elasticity for a group of commodities is equal to the weighted sum of the income elasticities of the individual commodities. The weights are equal to the expenditures laid out for the various commodities. This relation can be tested experimentally if one has at one's disposal the data of a household-budget inquiry. E_n can be calculated directly, and with the help of formula (1) Dr. Wold has performed these calculations on the data of a Swedish budget investigation. Moreover the n commodities were chosen so as to absorb practically the whole income. Therefore it could be concluded beforehand that E_n had to be ~ 1. The following results were obtained:

	Method of regression analysis		
	Elementary	Diagonal	Orthogonal
1. Calculated directly	0.88	0.92	0.92
2. Calculated as a weighted average	0.90	1.72	2.57

The good agreement between the two results obtained with the first elementary regression method is considered by Dr. Wold to be a strong argument in favor of this last method.

E. Schneider, Aarhus, *Price Policy in Periods of Depression.*—If an entrepreneur expects the price of his product not to rise above a certain loss-yielding price p during the next t time units (called days for shortness' sake), e.g., as a consequence of a decline of the business situation, he can choose between two alternatives: carrying on produc-

[1] "Efterpørselen efter melk in Norge," *Statsøkomisk Tidsskrift*, 1938, p. 1.

tion or closing down. Professor Schneider derived a formula[2] for the minimum price p_u, below which it is more profitable to stop production than to carry on business. Let x be the volume of sales per day; $k(x)$ total costs per day; and S_t the total costs involved in stopping production during t days (including costs of shutting down, the costs of upkeep during the shutdown, and the costs of bringing production back again at the old level). Now it is evident that it is more profitable to stop production as soon as:

$$S_t \leqq (k - px)t.$$

Therefore p_u will be given by:

$$S_t = (k - p_u x)t$$

or

$$p_u = \frac{k - (S_t/t)}{x}. \tag{1}$$

The curves A, given by $p = k(x)/x$, and B, given by (1), will divide the field of a quantity-price diagram into three parts:

1. The field above A. Every point in this field represents a situation yielding a profit.

2. The field between A and B. Every point represents a situation yielding a loss, which, however, is insufficient to justify stopping production.

3. The field between B and the x-axis. Every point represents a situation yielding a loss greater than the costs involved in shutting down production.

Generally (S_t/t) will decrease with increasing t and field 2 will become smaller. For very large values of t, (S_t/t) will practically be equal to zero and then we obtain the well-known result:

$$p_u \sim \frac{k(x)}{x} \qquad (t \to \infty).$$

For very long periods the price limit is given approximately by average cost. By dividing total costs into fixed and variable costs,

$$k(x) = f + v(x),$$

(1) can be transformed into

$$p_u = \frac{f - (S_t/t)}{x} + \frac{v(x)}{x}. \tag{2}$$

[2] This was first given by C. E. Schultz in *Annalen der Betriebswirtschaft*. Bd. 1, 1927, but not analyzed there.

Generally f will be $>(S_t/t)$, therefore

$$p_u > \frac{v(x)}{x}.$$

Generally the price limit will be higher than average variable cost. If $f=(S_t/t)$, p_u and $v(x)/x$ will coincide. Professor Schneider said that in 1931 he observed a case (in the German iron industry) in which $f<S_t/t$. There it was profitable to carry on production although even variable costs were no longer covered. For very short periods (S_t/t) will always be equal to f and then we come back again to the well-known theorem that production will be carried on as long as variable costs are covered.

During the discussion Professor THORKIL KRISTENSEN, Aarhus, remarked that the theory developed by Professor Schneider could be valid only for a monopolist, for, as soon as two or more producers are competing, other arguments, not considered by Schneider, will play a role, e.g., the risk that one's customers will be taken over by the competitors when production is stopped. Moreover it will be very difficult to estimate future prices with a sufficient degree of accuracy. Dr. ERLING PETERSEN, Oslo, pointed out that the difference between fixed and variable costs would become less important with increasing t. These last points were admitted by Professor Schneider.

The first part of the afternoon of August 26 was dedicated to problems of price policy. The chairman was Dr. John Einarsen, and the papers were as follows:

F. ZEUTHEN, Aarhus, *The Theory of Prices.*—Professor Zeuthen referred to an inquiry by Harrod, Hall, and Hitch,[3] proving that entrepreneurs generally fix their prices according to the "full cost" principle. Professor Zeuthen did not consider this result as being theoretically satisfying. According to him, the profit margin of the entrepreneur is determined by an arbitrary collective price policy. He underlined the importance of the publications of Winding Pedersen and Thorkil Kristensen, treating different forms of price policy and their consequences and showing that, even in the case of several competing enterprises, deviations from the liberalistic thesis may occur.

THORKIL KRISTENSEN, *Complex Monopoly.*—If a monopolist has a monopoly of two or more products at the same time, this is called a complex monopoly. A complex monopoly will lead to situations different from that existing when every product is sold by an individual monopolist only if these products interrelated. Two different forms are possible:

1. The same production factor is used in producing the products.
2. The demand functions for both the products are interrelated.

[3] *Oxford Economic Papers*, No. 2, May, 1939, pp. 1–45.

Only the last case will be considered. Restricting ourselves to two commodities A and B and denoting prices, quantities sold, and total production costs by p_a, p_b, x_a, x_b, and $v(x_a)$, $V(x_b)$, we may assume the following demand functions:

$$x_a = f(p_a, p_b), \tag{1}$$

$$p_b = F(p_a, p_b). \tag{2}$$

The monopoly profit will be given by

$$x_a p_a + x_b p_b - v(x_a) - V(x_b).$$

The maximum value of this expression will be obtained for the values of p_a and p_b which make the partial differential quotients with respect to p_a and p_b vanish. Introducing the symbols E_a, E_b, E_{ab}, and E_{ba}, defined by

$$E_a = -\frac{\partial x_a}{x_a} : \frac{\partial p_a}{p_a}, \qquad E_{ab} = \frac{\partial x_a}{x_a} : \frac{\partial p_b}{p_b},$$

and two analogous expressions, it is easy to verify that the maximum profit will be obtained if

$$p_a = \frac{v'(x_a)E_a x_a + x_b E_{ba}[p_b - V'(x_b)]}{x_a(E_a - 1)} \tag{3}$$

and an analogous expression for p_b. (1) can be transformed into:

$$p_a = M_a + \frac{x_b}{x_a}\,\frac{E_{ba}}{E_a - 1}\,[p_b - V'(x_b)], \tag{4}$$

where M_a is given by:

$$M_a = \frac{v'(x_a)E_a}{E_a - 1}. \tag{5}$$

Formally, M_a is equal to the expression obtained for p_a in the case of simple monopoly. [This may be seen by putting $E_{ba}=E_{ab}=0$ in (1)]. The numerical value of M_a, on the other hand, may be different in both cases, as the value of x_a (and of x_b) generally will be different in both situations. If, however, E_a is practically constant (independent of x_a and x_b), M_a will be the same in both cases and then the second term of the right-hand member of equation (4) may be considered as the difference between the prices of A with simple and complex monopoly. [Strictly speaking, it should also be assumed that $v'(x_a)$ is constant. This, however, seems to be reasonable as it implies that total

costs are a linear function of output.] From this term it is to be seen that the difference depends upon the product of three factors:

1. x_a/x_b, the relative importance of the outputs of the two commodities.

2. $E_{ba}/(E_a-1)$, the sensitivity of demand for B with respect to the price of A in relation to the sensitivity of the demand for A with respect to its own price.

3. $p_b-V'(x_b)$, the difference between price and marginal cost of B.

At the conclusion of Professor Kristensen's paper, an interesting discussion arose about the motives leading the entrepreneurs in fixing their output. According to Professor SCHNEIDER, the entrepreneurs do not always apply the principle of maximum profit. Professor KRISTENSEN pointed out that they always try to apply this principle but often do not reach their aim as a consequence of insufficient data. Dr. ERLING PETERSEN referred to the study of Dr. T. Koopmans, *Tanker Freight Rates and Tankship Building*, which suggests that the tanker pool might have reached better results by following a somewhat different policy.

WINDING PEDERSEN, Copenhagen, *Problems of Duopoly*.—The problem of monopoly can be solved very easily, but the problem of duopoly is indeterminate. The solution depends entirely on the assumptions made about the entrepreneurs' opinions about their mutual policy. The consequences of many schemes of hypotheses have already been worked out (Stackelberg, Kahn, and others) but these works do not contribute very much to a definite solution of the problem. According to Mr. Pedersen, such a solution can be given only by a dynamical theory. Only in such a theory due attention can be paid to influences from previous periods on the decision of today, whereas reactions of the purchasers to price changes can also be taken in consideration.

In the second half of the afternoon the following paper was presented:

A. BIJL, Amsterdam, *Wage Subsidies*.—Entrepreneurs in fixing their plans generally apply two different principles of "*rentabilité*."

1. The "choice principle," according to which out of different possibilities of production that one yielding the highest profit should be chosen.

2. The "principle of cost," according to which the expected return of a productive act should exceed its costs.

Of these principles only the first one is also in the interest of the State. Deviations from this principle will always mean wastage. The second principle, however, is of no interest to the State. Even loss-yielding production will generally increase the national income. Moreover, the policy derived from the principle is dependent on the division

of costs between the State and entrepreneurs, which is generally determined by pure convenience and historic growth, but for which general rules cannot be given. Therefore the rule of practical economic policy, that every production should carry its own costs, has no sound basis, part of the costs, (e.g., costs of using roads, etc.) always being paid by the State. In order to increase employment, the State often carries out public works, violating "the principles of costs." These works, however, include only a very small sector of production. A relatively larger increase of employment can only be obtained by breaking the limits given by the "principle of costs" for private production also. This cannot be reached by a general reduction of wages as the ratio of

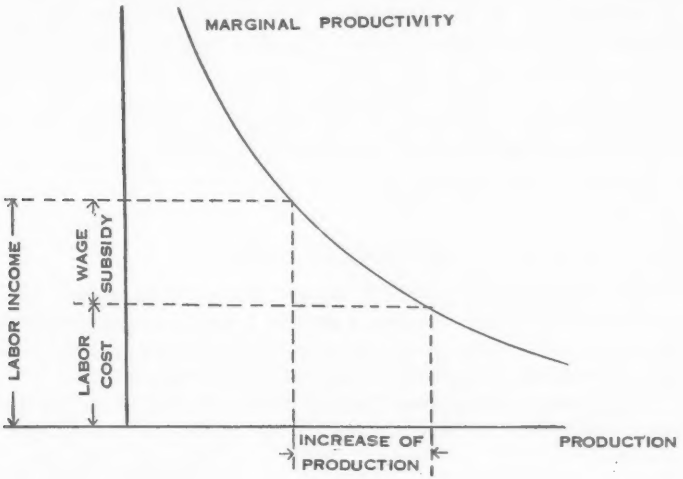

FIGURE 1

prices to wages tends to be independent of the money wage rate as is shown by theory and experience. The simplest possibility is a general subsidy to all entrepreneurs for wages for all workers, enabling the carrying out of production for which the value of the marginal output is smaller than the laborers' income. These wage subsidies would have to be paid out of higher taxes. Some figures were given showing that the same amount of money spent alternatively on public works and on wage subsidies in the Netherlands would probably yield the same increase of employment; the increase of national income, however, would be much larger in the latter case.

During the discussion Professor ZEUTHEN made two remarks:

1. There are different factors of production; wage reductions will lead to a shift to more labor-consuming production.

2. Wage subsidies continued for many years seem to be unnatural and will tend to increase the cost of administration.

Dr. BIJL replied that:

a. In the long run the prices of land and capital will also be determined by the wage rate.

b. Wage subsidies are necessary as long as full employment is not obtained automatically.

Professor PEDERSEN did not see any difference between giving no subsidies, and giving subsidies and taking back the same amount by taxes. Moreover he asked why all wages should not be paid in the form of subsidies. Dr. BIJL's answer may best be illustrated by Figure 1, representing the marginal productivity of a function of production. Since production will be carried on as long as marginal production exceeds labor costs (labor income-wage subsidies) a wage subsidy will yield an increase of production. The subsidy has to be fixed so as to increase production until full employment is reached.

The Hague, The Netherlands

EDITORIAL NOTE

Since communications with Professor Ragnar Frisch, Editor, were interrupted for several weeks in April and May, the responsibility for the acceptance of most of the papers published in this issue has been taken by Professor Joseph A. Schumpeter, Associate Editor.

A cable was received from Professor Frisch on May 23 stating that postal service was resumed and that he was sending on manuscripts.

MANAGING EDITOR